Report No. DOT/FAA/RD-81/60
DOT/FAA/CT-81/15

ANALYSIS OF A NONLINEAR ALTITUDE TRACKING METHOD

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FINAL REPORT

NOVEMBER 1981

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Prepared for

U. S. DEPARTMENT OF TRANSPORTATION
FEDERAL AVIATION ADMINISTRATION
Systems Research & Development Service
Washington, D. C. 20590

NOTICE

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT/FAA/RD-81/60	AD-A108 828	
4. Title and Subtitle: ANALYSIS OF A NONLINEAR ALTITUDE TRACKING METHOD		5. Report Date: November 1981
		6. Performing Organization Code
7. Author(s): Barry R. Billmann		8. Performing Organization Report No.: DOT/FAA/CT-81/15
9. Performing Organization Name and Address: Federal Aviation Administration Technical Center Atlantic City Airport, New Jersey 08405		10. Work Unit No. (TRAIS)
		11. Contract or Grant No.: 052-242-320
12. Sponsoring Agency Name and Address: U.S. Department of Transportation Federal Aviation Administration Systems Research and Development Service Washington, D.C. 20590		13. Type of Report and Period Covered: Final September to December 1980
		14. Sponsoring Agency Code: ARD-200
15. Supplementary Notes		

16. Abstract

This report analyzes the performance of the nonlinear altitude tracker developed for the Active Beacon Collision Avoidance System (BCAS) by Lincoln Laboratory. The tracker is intended for use in the collision avoidance logic of the BCAS system. The nonlinear tracker performance was characterized through comparisons with the previous altitude tracker. The original tracker, used by the collision avoidance logic, was a simplistic Alpha-Beta ($\alpha - \beta$) tracker.

The nonlinear tracker performance evaluation was conducted in three phases: (1) The stand-alone error characteristics of the tracker were obtained. Simulated mode C report sequences were provided directly to the tracker. (2) The nonlinear tracker was integrated directly into the collision avoidance logic. With the use of the Fast-Time Encounter Generator (FTEG), a comparative study of performance with the nonlinear tracker versus the $\alpha - \beta$ tracker was made. (3) Selected live flight test encounters were used to analyze the relative performance of the $\alpha - \beta$ tracker versus the nonlinear tracker.

The stand-alone analysis revealed that the nonlinear tracker consistently had smaller maximal errors in vertical rate estimation and a smaller transient rate response delay than did the $\alpha - \beta$ tracker. Both the live flight test encounter simulations and the FTEG scenario simulation indicated that nonlinear tracking often caused an increase in separation for encounters with vertically accelerating threats and reduced occurrences of incorrect command sense choice.

17. Key Words	18. Distribution Statement
Nonlinear Altitude Tracker Alpha-Beta Tracker Fast-Time Encounter Generator (FTEG) Beacon Collision Avoidance System (BCAS)	Document is available to the U.S. public through the National Technical Information Service, Springfield, Virginia 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	51	

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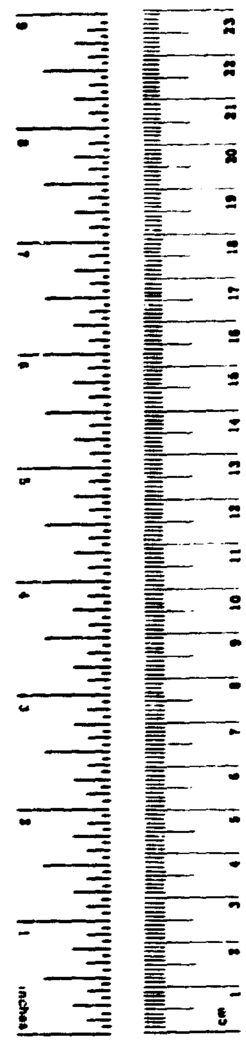
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

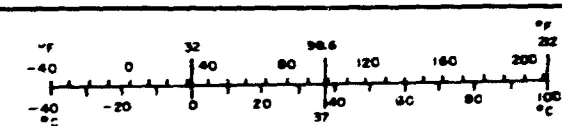
Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



ACKNOWLEDGEMENTS

The author wishes to express appreciation to Richard Cleary of the Federal Aviation Administration Technical Center. Mr. Cleary was instrumental in identifying flight test Active Beacon Collision Avoidance System tracking anomalies discussed in this report. The author also wishes to express appreciation to John Andrews of Lincoln Laboratory, the developer of the nonlinear tracker analyzed in this report. Mr. Andrews showed extreme patience in listening to and answering questions about the tracker.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (Continued)

LIST OF TABLES

INTRODUCTION

PURPOSE.

The purpose of this report is to describe the analysis of the nonlinear tracker algorithms developed by Lincoln Laboratory (reference 1). The nonlinear tracker was developed to replace the current alpha-beta ($\alpha - \beta$) tracker in the Active Beacon Collision Avoidance System (BCAS) intruder and own aircraft tracking logic (reference 2). The report documents the performance of the nonlinear tracker in terms of vertical rate error characteristics and vertical separation at the closest point of approach (CPA) following BCAS commands.

SCOPE.

This report is intended to characterize the impact of the mode C altitude report quantization on vertical rate tracker estimates. This report presents results of fast-time simulations of nonlinear tracker performance.

An analysis was made of the stand-alone vertical rate and position error characteristics and comparison with the performance of the $\alpha - \beta$ vertical tracker using specially developed conflict scenarios and encounters based on live BCAS flight test data.

BACKGROUND.

The BCAS logic requires the accurate sequential estimation of own aircraft's and the intruder's vertical position and vertical rate. This information is generated every logic cycle (nominally every second) in the own aircraft and intruder aircraft tracking modules. In the presence of missing target reports, the tracking modules can coast the vertical position and vertical rate based on previous track history. On every logic cycle after the logic has updated own and intruder aircraft tracks, the vertical position and rate estimates are used to perform up to three vital collision resolution functions.

The first function is to determine which intruders are threats and if a resolution advisory is required. Previous testing of the BCAS logic showed that the errors associated with the $\alpha - \beta$ tracked vertical position and vertical rate only caused a 1- to 5-second variation in the time at which the BCAS command first appeared (reference 3).

The second use is to determine the sense of escape maneuver. Active BCAS can only provide escape resolution in the vertical domain. The sense of maneuver is selected by modeling an own aircraft response to, first, a climb command and, then, a descent command. The sense of maneuver which would provide the greater separation at CPA is then selected. Sense selection is made on the first logic cycle that the intruder is declared a threat. Once selected, the sense of command for a specific intruder does not change throughout the encounter.

Since the sense is selected only once, it is imperative that the occurrences of incorrect sense choices be minimized. To determine the threat's vertical position at CPA, the BCAS logic projects the threat's vertical position at time of detection. The projection, which is based on the tracked vertical rate of the intruder at time of detection, may entail as much as a 35-second position projection. As a result, the peak error in the tracked vertical rate at time of detection can be multiplied by 35 in the determination of the threat's projected vertical position at CPA. During previous testing of the BCAS logic, it was discovered that the peak errors in the vertical rate estimate provided by the $\alpha - \beta$ tracker often resulted in an incorrect sense choice and marginal separation performance (reference 8).

The third use of the tracked vertical position and rate is made in the determination of the severity of the BCAS command. The logic selects one of three types of maneuvers: positive commands (climb or descend); negative commands (do not descend or do not climb); or vertical speed limit commands (limit descent rate or limit climb rate). The positive commands are the most severe, since they require a positive response by the BCAS aircraft to generate separation. Negative and limit commands may not require a change in the BCAS aircraft's vertical rate in order to provide proper separation. Unlike the sense of the command, command severity can transition during an encounter period. The only requirement is that, once a particular command severity is selected, it must be displayed for at least 5 seconds prior to changing.

Logic testing has not detected a command severity selection problem that can be traced to errors in vertical rate estimation. The major impact on large performance caused by errors in vertical rate estimation occurs in the area of sense selection.

DISCUSSION

PREVIOUS VERTICAL TRACKER.

The inputs to the BCAS vertical tracking logic are the mode C altitude reports. These reports provide altitude information to the nearest 100 feet. The ideal vertical tracker must provide accurate position and rate estimates in spite of the 100-foot granularity in the input data. Through optimization of the α (altitude tracking constant), and β (rate tracking constant) parameters, the $\alpha - \beta$ tracker attempts to provide smooth vertical rate estimates without excessive delays in detecting rate changes. The $\alpha - \beta$ vertical tracker used by the Active BCAS logic originated with the intermittent positive control (IPC) collision avoidance algorithms (reference 4).

With consistent 1-second updates, the vertical position estimate of the α-β tracker at time t is $Z(t)$ where

$$Z(t) = (1-\alpha)*(Z(t-1) + \dot{Z}(t-1)) + \alpha*Z_r(t) \tag{1}$$

where $\dot{Z}(t)$ and $Z_r(t)$ are the rate estimate and mode C report at time t. The altitude tracking constant $\alpha = 0.4$.

The vertical rate estimate at time t is $\dot{Z}(t)$ and is obtained as follows:

$$\dot{Z}(t) = (1-\beta)*\dot{Z}(t-1) + \beta*(Z_r(t) - Z(t-1)) \tag{2}$$

where the original altitude rate tracking constant was $\beta = 0.15$.

During periods of missing data reports (up to 10 seconds) (1) is replaced by

$$Z(t) = Z(t-\Delta t) + \dot{Z}(t-\Delta t)*\Delta t \tag{3}$$

where the duration of the missing data period is Δt.

During this period, the rate estimate is a constant, $\dot{Z}(t-\Delta t)$. The initial evaluation of the BCAS collision avoidance logic indicated that the probability of incorrect sense choices for threats maneuvering at constant vertical rates was significant. These sense choice problems were traced to the errors in the rate estimate at the time of threat detection (references 5 and 6). Since the original β parameter value (0.15) was based on the 4.7-second update rate of IPC, the presence of the 1-second

update rate of Active BCAS caused large quantization induced errors in the rate estimate. In turn, the projection of these large rate errors for up to 35 seconds caused an incorrect relative position projection at CPA.

Following a study by Broste (reference 7), the β parameter was reduced from 0.15 to 0.10. This change caused the rate estimate to be more heavily smoothed by previous track history, at the cost of a slight loss in the tracker's transient rate response time. Reference 7 demonstrates a significant reduction in the root mean square error in the rate estimate when β is reduced. However, it is important to note that since the sense selection is the result of the stochastic condition of the rate estimate at a single instant in time, it is the control of the maximal error in the rate estimate that should be of primary concern. With $\beta = 0.10$, the cyclic errors in the rate estimate and the possible error in the vertical position projection for a constant 500 feet per minute (ft/min) vertical maneuver are shown in figure 1. The mode C reports are also shown in figure 1.

At a constant 500-ft/min vertical rate, the mode C report will change once every 12 seconds. The peak quantization-induced error in the rate estimate exceeds 8 feet per second (ft/sec). This is the magnitude of the vertical rate (500 ft/min = 8.33 ft/sec). The peak error in the rate estimate occurs 2 seconds after the mode C report changes. The error in the rate estimate results in a cyclic error in the position projection. During the 12-second tracking error cycle, the error in the projected position exceeds 200 feet, six times.

Similar analysis of the cyclic errors in the rate estimate caused by quantization was performed for constant rate maneuvers between 500 and 3,500 ft/min in increments of 100 ft/min. The peak error in the rate estimate and the proportion of time the projected position error exceeded 200 feet are shown in table 1.

Table 1 indicates that the largest percentages of projected position error are associated with vertical rates between 500 and 1,100 ft/min. Nonzero probabilities of the projection error exceeding 200 feet appear again between 1,600 and 1,900 ft/min and between 2,700 and 3,200 ft/min. Rates in the range of 1,600 to 1,900 ft/min are characterized by time intervals between mode C transitions which (when quantized in seconds) do not result in constant time measurements between mode C report transitions. At 1,500 ft/min, the mode C report changes every 4 seconds; it changes every 3 seconds at 2,000 ft/min. Therefore, rates between 1,500 and 2,000 ft/min cause mode C report transitions to cycle between 3 and 4 seconds. It is this pattern of quantized time measurements which are not constant that cause the peak errors in the rate estimate. The same is true in the 2,700- to 3,200-ft/min range. Except at the 3,000-ft/min rate where the transition period is a constant 2 seconds, each of the rates represent nonconstant transition periods. For 2,700-to 2,900-ft/min rates, the transition period cycles between 2 and 3 seconds. For rates above 3,000 ft/min, it cycles between 1 and 2 seconds.

INITIAL FLIGHT TEST RESULTS.

Initial live flight tests were conducted with an $\alpha - \beta$ tracker performing the internal BCAS logic vertical tracking function. The parameter values were $\alpha = 0.4$ and $\beta = 0.1$. Almost immediately, problems in sense choice were detected for intruders which were essentially in level flight. The sense choice problems were traced to mode C report boundary transitions. The patterns of mode C reports caused by the boundary transitions are called mode C excursions.

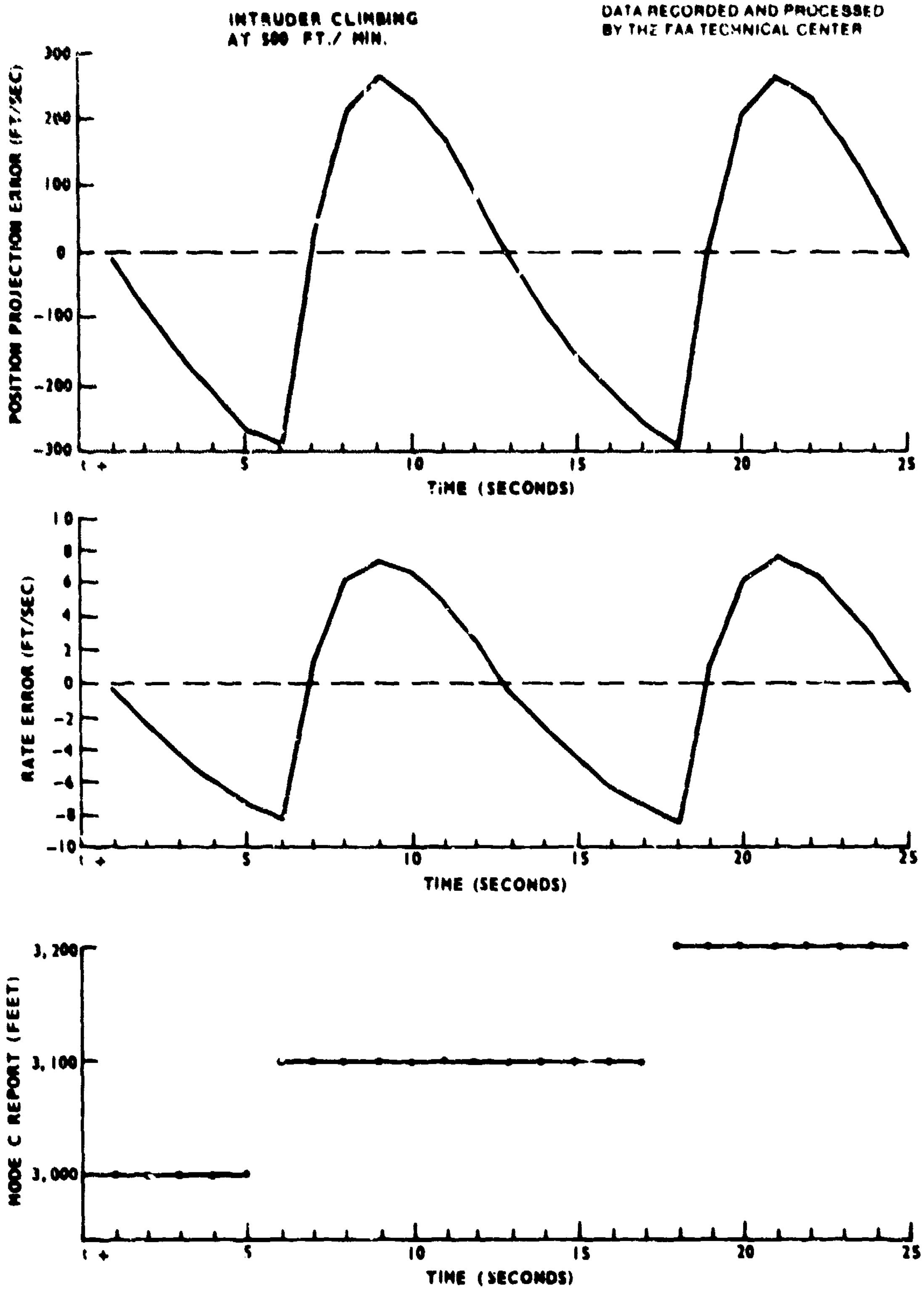


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FIGURE 1. RATE AND POSITION 35-SECOND PROJECTION ERROR CYCLES (β = 0.10)

DATA RECORDED AND PROCESSED
BY THE FAA TECHNICAL CENTER

TABLE 1. QUANTIZATION-INDUCED CYCLIC PEAK ERROR IN THE RATE ESTIMATE AND THE IMPACT ON THE PROJECTED POSITION $\beta = 0.10$

RATE (ft/min)	PEAK ERROR (ft/sec)	PROPORTION OF TIME PROJECTED POSITION ERROR EXCEEDS 200 FT
500	8.15	0.50
600	7.96	0.30
700	8.31	0.23
800	8.03	0.06
900	7.35	0.09
1,000	5.79	0.17
1,100	6.81	0.09
1,200	4.90	0.00
1,300	6.57	0.07
1,400	4.67	0.00
1,500	3.92	0.00
1,600	5.72	0.07
1,700	6.73	0.05
1,800	4.90	0.00
1,900	6.55	0.09
2,000	2.84	0.00
2,100	4.74	0.00
2,200	5.62	0.00
2,300	5.08	0.00
2,400	3.19	0.00
2,500	4.50	0.00
2,600	3.82	0.00
2,700	5.97	0.06
2,800	6.41	0.07
2,900	8.23	0.07
3,000	1.61	0.00
3,100	8.22	0.07
3,200	6.40	0.08
3,300	4.83	0.00
3,400	6.21	0.02
3,500	4.99	0.00

Examples of mode C excursions seen in live flight testing are shown in figure 2.

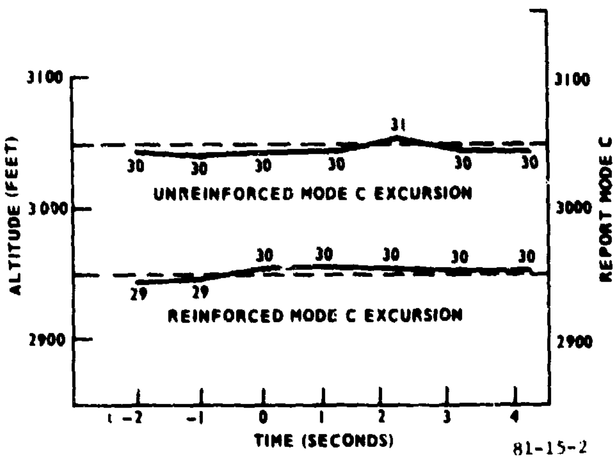

FIGURE 2. LEVEL FLIGHT MODE C REPORT EXCURSION PATTERNS

In the first case, the intruder is in level flight near the mode C transition boundary of 3,050 feet. A slight deviation in the flight path takes the aircraft above the transition boundary causing a 100-foot change in the quantized altitude report. If the aircraft returns to below 3,050 feet on the next cycle, the 3,100-foot mode C report is not reinforced. On the other hand, the aircraft can remain above 3,050 feet and the 3,100-foot mode C report can be reinforced. The quantization-induced rate errors for these two mode C excursions are shown in figure 3.

Although the intruder's flight path is nearly level, the slight altitude deviations across the mode C transition boundary can induce significant errors in the rate estimate. For the unreinforced excursion, the peak error of 10 ft/sec occurs when the mode C report changes. The rate error decays once the mode C report returns to its previous value. The peak error of 10 ft/sec could cause a 350-foot error in the projected vertical position. In the unreinforced case, the error in the rate estimate is depressed below the threshold for a 200-foot error in projected position 1 second after the initial excursion.

For cases where the aircraft remains above the transition boundary reinforcing the mode C excursion, the peak error in the rate estimate occurs 2 seconds after the mode C transition. The peak error of 16.5 ft/sec could cause the error in projected position to exceed 575 feet. When the aircraft remains above the transition boundary, the induced errors in the rate estimate remain above the 200-foot position error threshold for 10 seconds. This analysis reveals that with $\alpha - \beta$ tracking ($\alpha = 0.4$ and $\beta = 0.1$) mode C quantization-induced rate errors can remain above the rate threshold that causes 200-foot errors in position projection for 1 to 10 seconds, depending on the duration of the mode C excursion.

Both mode C excursion patterns investigated occurred often in flight testing. Logic changes were made to control the impact of the induced rate error caused by mode C excursions. Without abandoning $\alpha - \beta$ tracking, the following CAS logic changes were made:

1. In order to make the tracker less sensitive to mode C transitions, the rate tracking constant β was reduced to 0.05.

2. The sense choice logic became a "balanced" logic, in the sense that neither a climb nor descent avoidance maneuver was favored. Previously, sense choice logic had favored the selection of descent sense maneuvers.

3. A rate clip parameter, ZDLVL, was added to the sense choice logic. If the rate estimate was less than 10 ft/second any sense selection that might occur on that logic cycle was based strictly on relative vertical position.

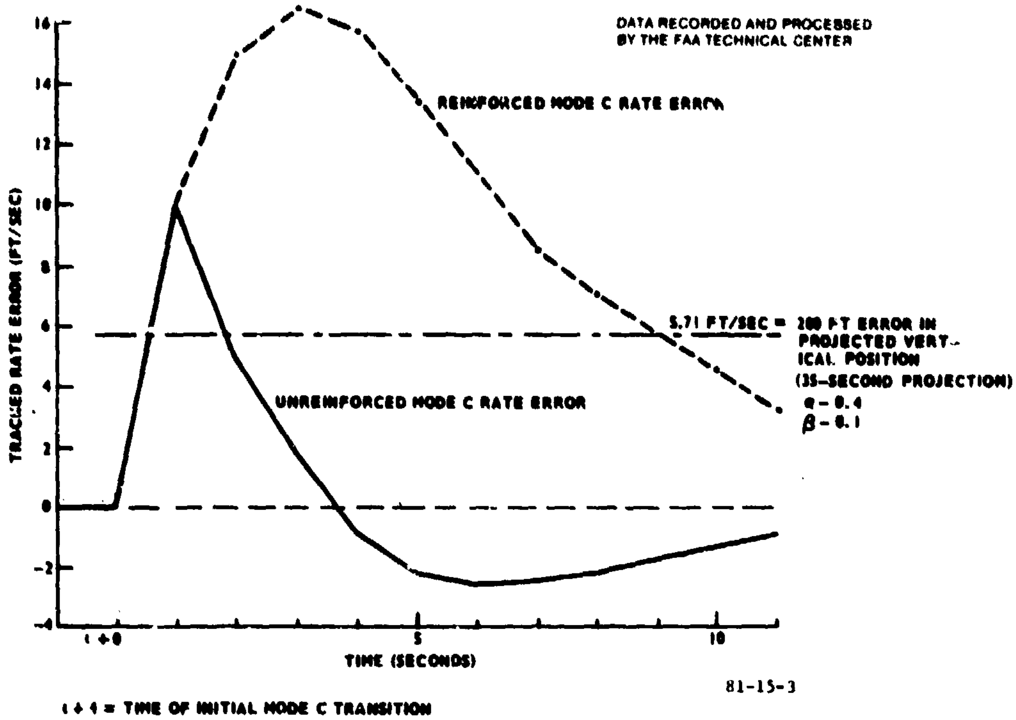

FIGURE 3. MODE C EXCURSION-INDUCED TRACKED RATE ERRORS

Subsequent flight testing verified that these logic changes eliminated sense choice problems associated with mode C excursions for aircraft essentially in level flight. However, the reduction of β to 0.05 has significantly reduced the tracker's acceleration detection capabilities. Broste (reference 7) uses the term "tracker equivalent memory" to characterize the tracker's transient rate response time as a function of α and β. Tracker equivalent memory approximates the response time of an α - β tracking system to input transitions and identifies the tracker's ability to detect accelerations. Table 2 shows the cost incurred in terms of acceleration detection with the reduction of β to 0.05.

TABLE 2. TRACKER RESPONSE DELAY AS A FUNCTION OF β

Rate Tracking Constant β	Tracker Response Delay-Equivalent Memory (Seconds)
0.15	6.87
0.10	9.06
0.05	14.46

The increase in smoothing of the rate estimate with the reduction of β to 0.05 from 0.15 has resulted in the doubling of the tracker's transient rate response delay. This delay, during vertical accelerations by intruder aircraft, could critically impact BCAS threat detection by inducing delayed alarms causing insufficient separation.

NEW NONLINEAR TRACKING APPROACH.

In order to provide for smoother rate estimates without causing a significant increase in the tracker's transient rate response delay for certain conditions, Lincoln Laboratory has developed a new nonlinear tracking scheme. The tracker, which is considerably more complex than the $\alpha - \beta$ tracker, makes greater use of the mode C report history than the $\alpha - \beta$ tracker. The description of the tracker is presented in appendix A. The remaining portions of this report present the results of the comparison of the nonlinear tracking performance with the $\alpha - \beta$ tracking performance ($\alpha = 0.4$, $\beta = 0.05$).

RESULTS AND ANALYSIS

The analysis of the nonlinear tracker performance was conducted in three phases. The first characteristic analyzed was the maximal error in the rate estimate for a given constant rate vertical maneuver. To avoid the influence of tracker initialization procedures, the trackers were exercised with the fixed vertical rate mode C inputs until the trackers reached steady state conditions. The resulting tracker steady state conditions may contain oscillations in rate estimation. These oscillations are called limit cycle oscillations (reference 9) and appear in a repeatable pattern. A comparison is made between the maximal errors in the limit cycle oscillations for each tracker. In this phase, mode C altitude reports were provided directly to the nonlinear and $\alpha - \beta$ tracking logic. In the second phase, each tracker was interfaced into the Active BCAS logic; performance was analyzed through simulation using the Fast-Time Encounter Generator. In the final phase, selected live flight test data were used to compare the $\alpha - \beta$ and nonlinear tracker's performance.

STAND-ALONE STEADY STATE PERFORMANCE.

In a steady state 500-ft/min climb, the mode C report changes every 12 seconds. Figure 4 presents the comparison of the limit cycles for both trackers. This causes the periodicity of rate error cycle to equal 12 seconds. The peak error in the rate estimate within that cycle are the errors which are compared. Figure 5 presents the comparison of the peak rate error for both trackers for rates ranging from 100 to 3,500 ft/min. The rate error which would yield a 100-foot vertical position projection error (2.86 ft/sec) is also identified.

In no case does the steady state nonlinear tracker peak rate error exceed the $\alpha - \beta$ tracker's peak rate error for rates less than 400 ft/min. Nominal errors in the rate estimates which occur for both trackers are large. However, the magnitude of the rate estimates for both trackers is less than 10 ft/sec. As a result, the BCAS logic assumes the aircraft is essentially level and ignores the rate estimate in command sense selection. Current relative position is used instead of projected vertical position to determine command sense.

The altitude bin occupancy time is defined as the duration of a constant mode C report. For a 500-ft/min rate, the bin occupancy time is 12 seconds. When the bin occupancy time is constant, the nonlinear tracker's peak rate error is 0 ft/sec. Looking at figure 5, this condition occurs for rates of 400, 500, 600, 1,000, 1,200, 1,500, 2,000, and 3,000 ft/min. This occurs because these

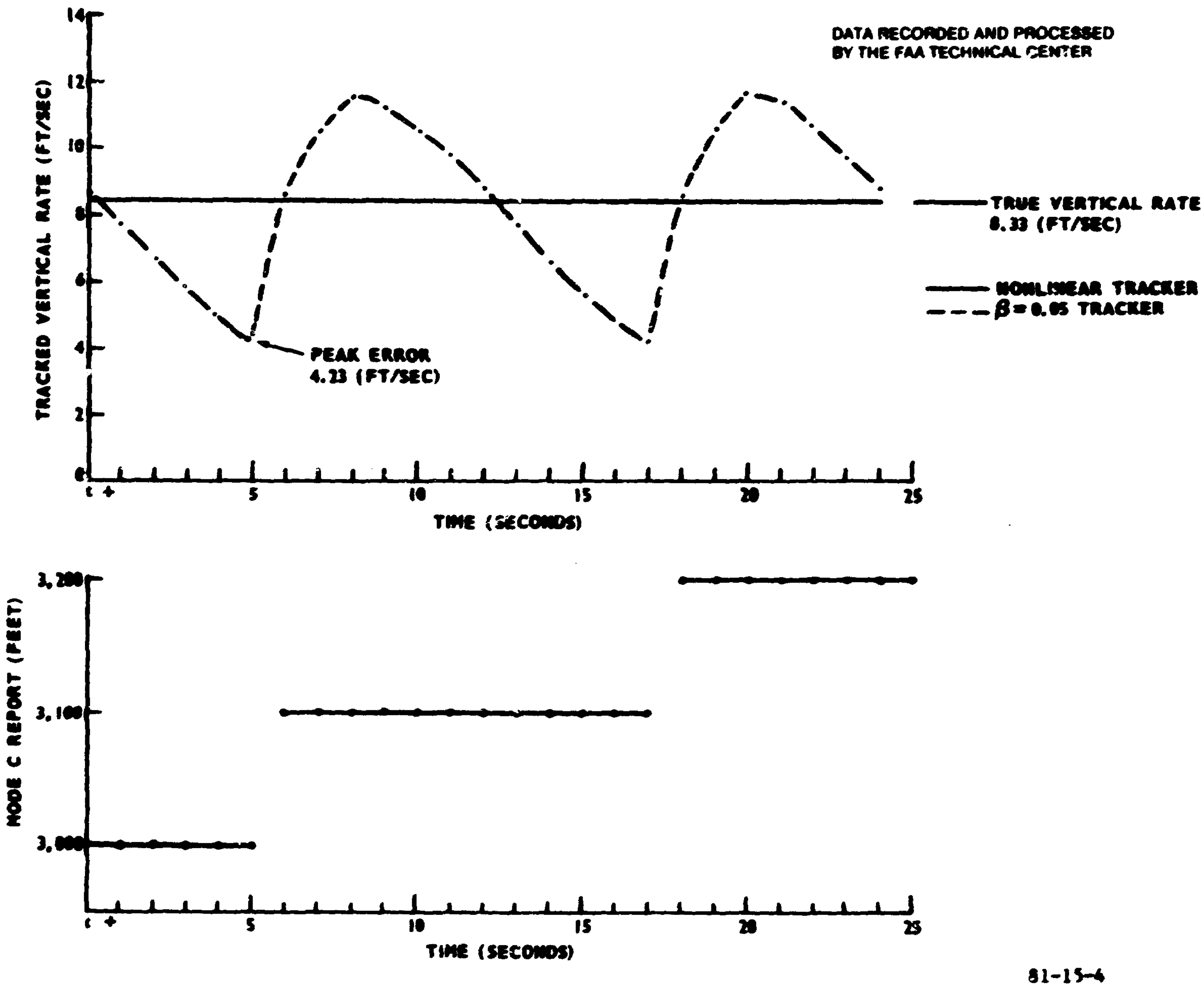

FIGURE 4. COMPARISON OF LIMIT CYCLE OSCILLATION FOR 500 FT/MIN CLIMB

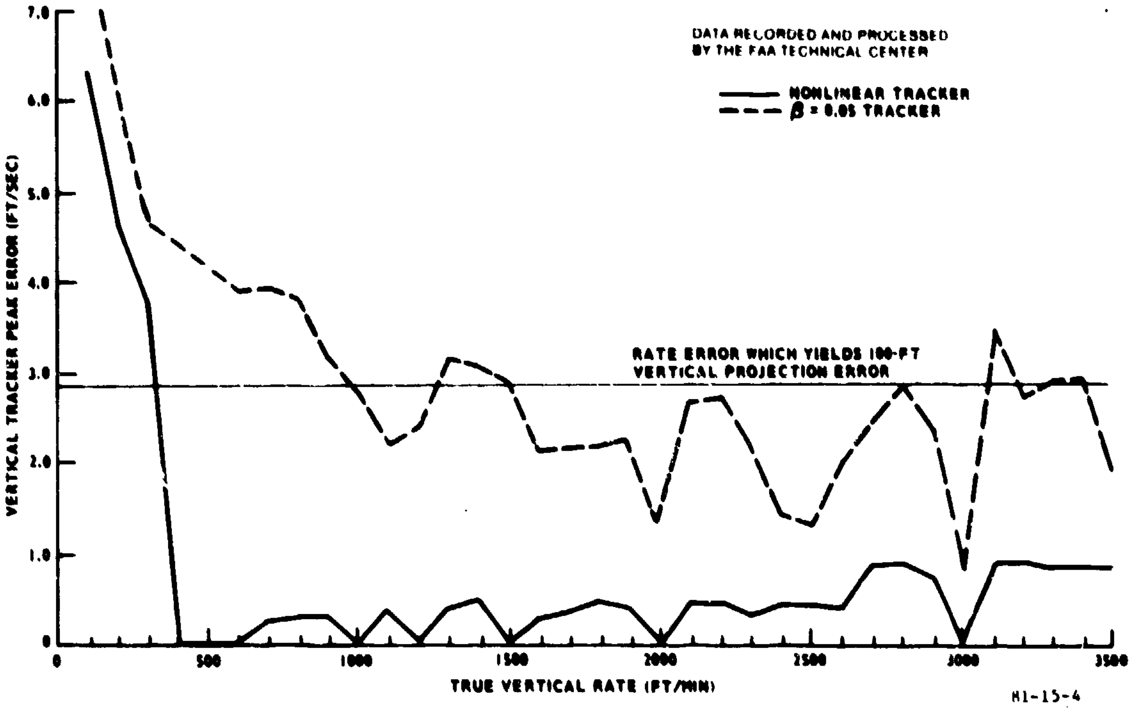

FIGURE 5. MAXIMAL ERROR COMPARISON FOR FINAL STEADY STATE CLIMB

rates represent constant integer bin occupancy times of 15, 12, 10, 6, 4, 3, and 2 seconds, respectively.

The nonzero peak errors in the nonlinear tracker occur for rates which do not yield a constant bin occupancy time. For instance, a 2,700-ft/min rate (45 ft/sec) yields the following bin occupancy sequence: 3 seconds, 2 seconds, 2 seconds, 2 seconds, 3 seconds. The nonconstant bin occupancy times cause the rate estimate errors.

Regardless of the consistency of the bin occupancy time, the α - β tracker always has a nonzero peak rate error. Both trackers have mean rate errors of 0 ft/sec for all steady state constant rate conditions analyzed. It is important to note that the constant rate vertical maneuvers usually employed by pilots (500, 1,000, 1,500, and 2,000 ft/min) all represent constant bin occupancy times. The nonlinear tracker, in these cases, has the capability of providing error-free rate estimates.

ACCELERATION PERFORMANCE.

The stand-alone analysis was extended to characterize tracker performance during and following vertical accelerations. In the first case analyzed, mode C inputs represented a level flight aircraft that began 8 ft/sec^2 (0.25 g) acceleration at time t. The acceleration continued until a final vertical rate was achieved. The analysis was designed to compare the difference in the time required for the trackers to obtain an accurate estimate of the final vertical rate. Conservative estimates of the time delay were obtained by initially having the level flight mode C inputs occur while the aircraft was at the lowest portion of the altitude bin. In this way, with an acceleration into

the climb condition, the longest period would expire prior to a mode C report transition.

In figure 6, the final rate investigated was 1,000 ft/min. This final rate is achieved 2.07 seconds after the acceleration begins. The first mode C transition is observed at t+7. Rate estimate for the $\alpha - \beta$ tracker is 6.48 ft/sec. The nonlinear tracker considers this first change in mode C to be an isolated change since the previous mode C reports had indicated level flight. As shown in appendix A, the nonlinear rate estimate at t+7 is 8 ft/sec, the value of parameter P1. The next mode C transition occurs at t+13. Between t+7 and t+13, the $\alpha - \beta$ rate estimate peaks at t+10. This corresponds to the occurrence of the peak rate estimate for reinforced mode C excursion rate estimates as shown in figure 3. After t+13, the nonlinear tracker has observed the constant bin occupancy time for the 1,000-ft/min rate (6 seconds), and error-free rate estimates occur. The $\alpha - \beta$ rate estimates show the 6-second cyclic pattern associated with the bin occupancy time. The $\alpha - \beta$ rate estimate converges to 1,000 ft/min but will oscillate around that value indefinitely. For 0.25 g acceleration and final rate of 1,000 ft/min, both the tracks experience the same delay in providing accurate rate estimates (13 seconds). The advantage of the nonlinear tracker is the error-free condition that can exist after t+12.

Figure 7 depicts the comparison when the final rate is increased to 2,000 ft/min. The final rate is achieved in 4.15 seconds. The first mode C transition occurs at t+6. Again, this initial transition is interpreted as an isolated transition by the nonlinear tracker. The same rate estimate (8 ft/sec) occurs as it occurred for the initial transition in the 1,000 ft/min case. The second transition occurs at t+9 and reflects the constant 3-second bit occupancy time. For times greater than t+9, considerable differences in the rate estimates provided by the two trackers exist. The cyclic 3-second pattern of rate estimates are apparent in the $\alpha - \beta$ rate estimate. Accurate estimates of the rate occur 10 seconds earlier with the nonlinear tracker. The error in the $\alpha - \beta$ rate estimate exceeds 10 percent of the actual rate until t+19. This delay in obtaining an accurate estimate of the final rate is due to the large equivalent memory (table 2) associated with the $\beta = 0.05$ value.

In figure 8, results for a final rate of 3,000 ft/min (2-second bin occupancy time) are shown. In this case, the final rate is achieved in 6.22 seconds. The first mode C transition occurs at t+5 and the second at t+7. During the acceleration phase, between t and t+6, both trackers provide very poor rate estimates. However, following the second transition at t+7, the nonlinear tracker provides an error-free rate estimate. Due to the equivalent memory of the $\alpha - \beta$ tracker, accurate estimates of the true rate, 50 ft/sec, do not occur until t+18. Hence, accurate rate estimates occur 11 seconds sooner with the nonlinear tracker.

A comparison of tracker performance for slow accelerations was also made. The case where the acceleration rate is 0.1 g (3.21 ft/sec^2) and the final rate is 2,300 ft/min will be reviewed. When the final rate is 2,300 ft/min, the bin occupancy times are not constant. The bin occupancy pattern is a combination of 2- and 3-second occupancy times. The results of the comparison are shown in figure 9. Achievement of the final rate required 11.92 seconds. Because of the slow acceleration, the first transition does not occur until t+8. The second transition does not occur until t+11. Although the nonlinear rate error exceeds 4 ft/sec between t+11 and t+15, the error is less than one-half the error for the $\alpha - \beta$ tracker for the same time period.

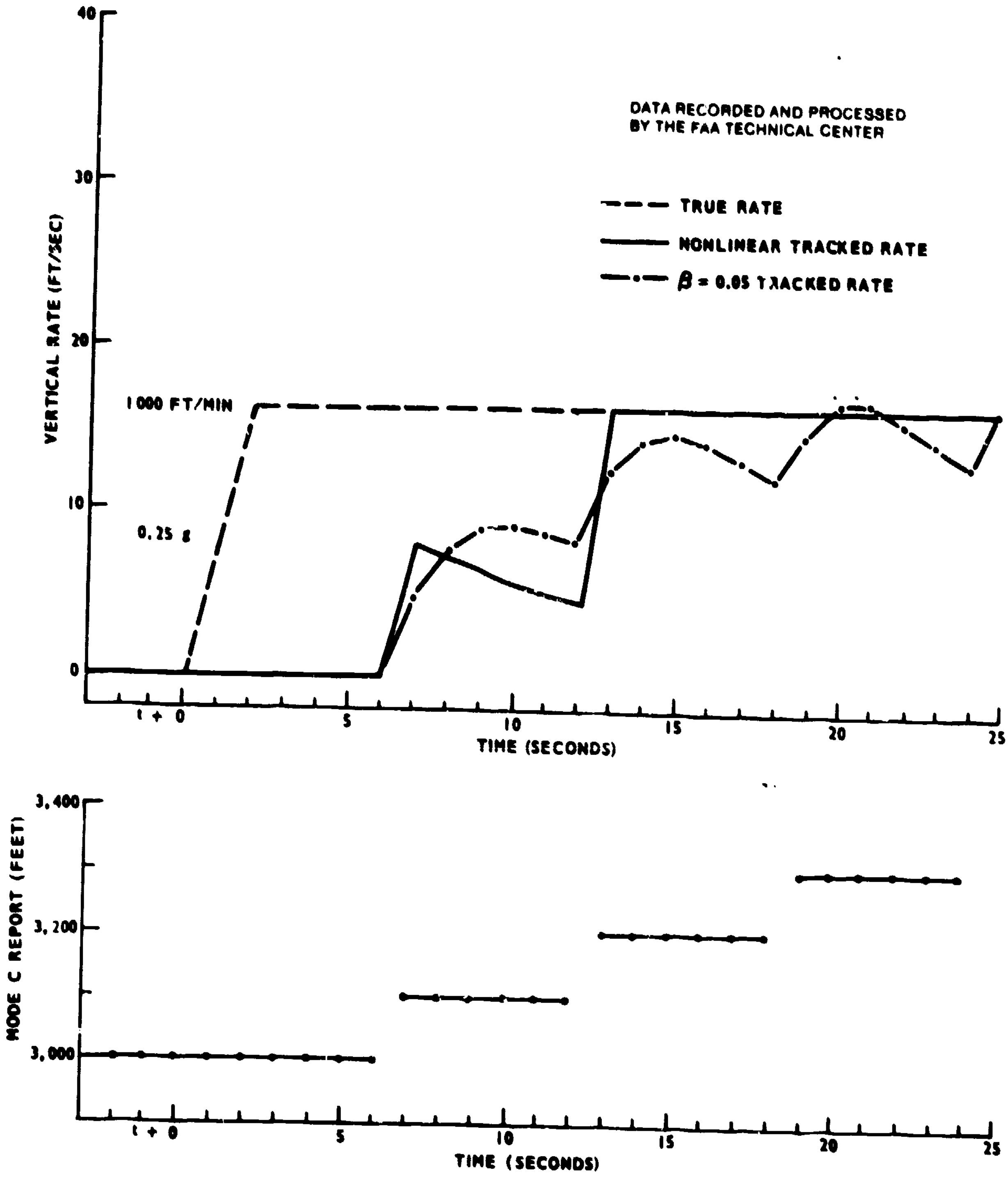

FIGURE 6. COMPARISON OF ACCELERATION PERFORMANCE — ACCELERATION RATE = 0.25 g, FINAL RATE = 1,000 FT/MIN

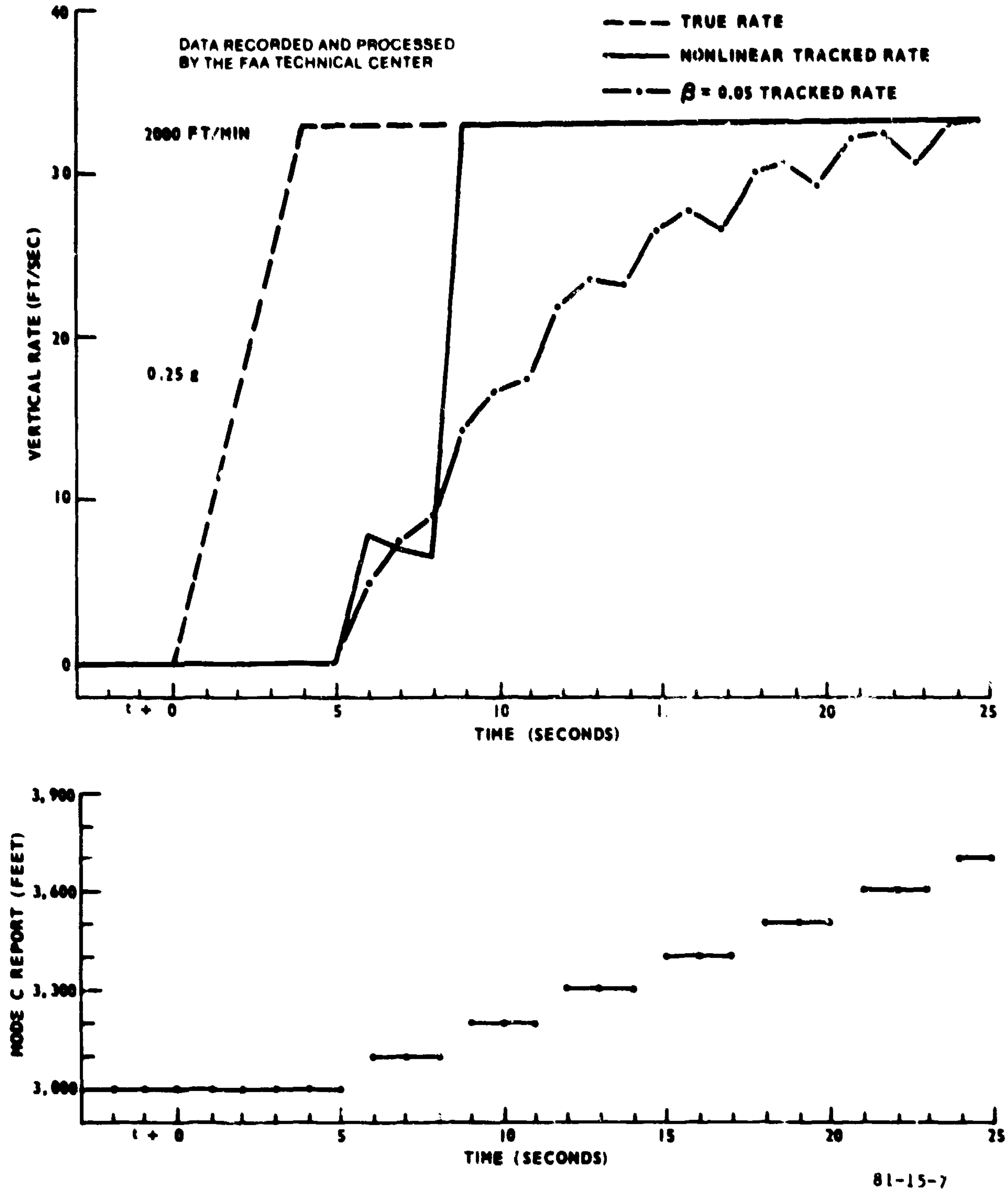

FIGURE 7. COMPARISON OF ACCELERATION PERFORMANCE — ACCELERATION RATE = 0.25 g, FINAL RATE = 2,000 FT/MIN

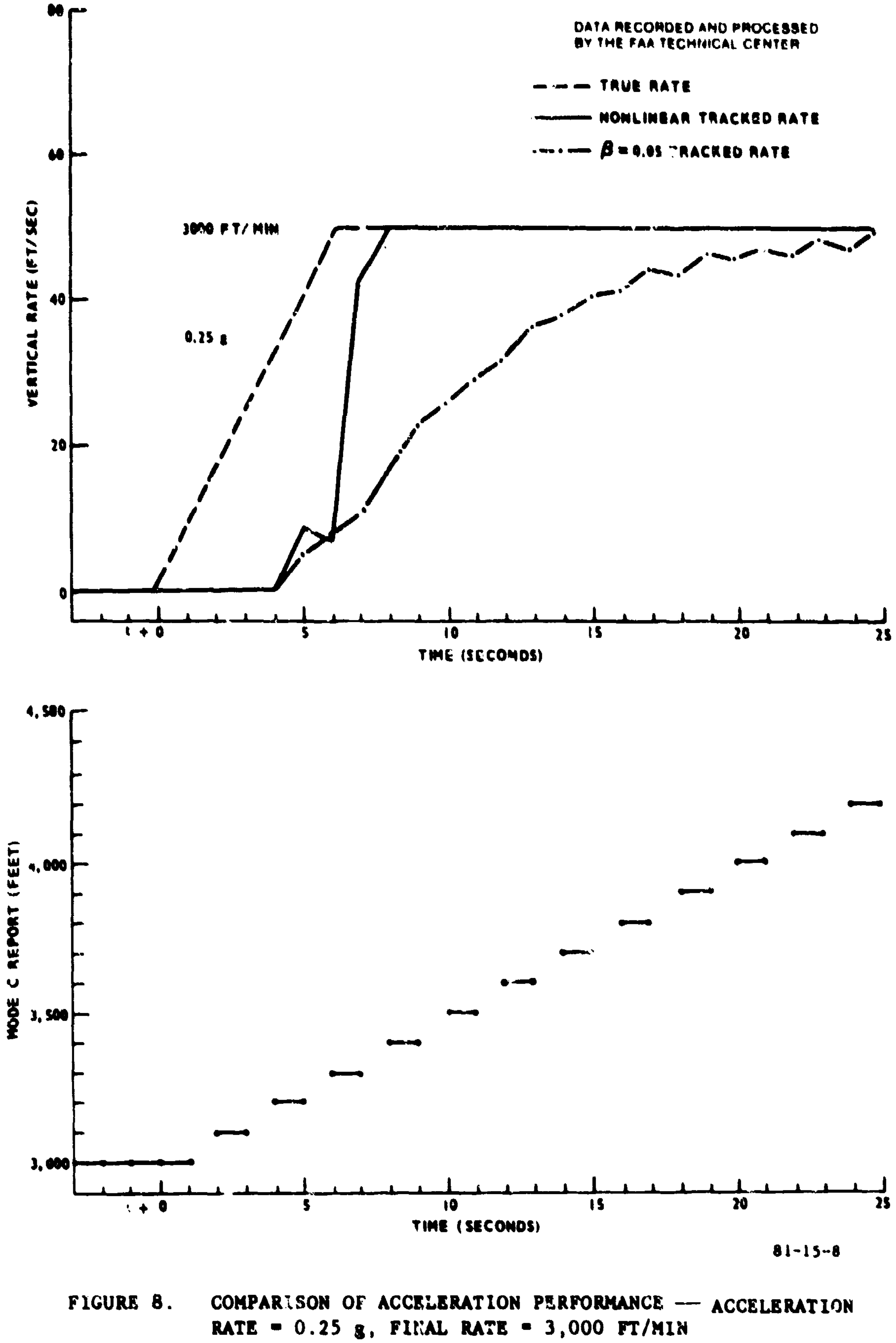

FIGURE 8. COMPARISON OF ACCELERATION PERFORMANCE — ACCELERATION RATE = 0.25 g, FINAL RATE = 3,000 FT/MIN

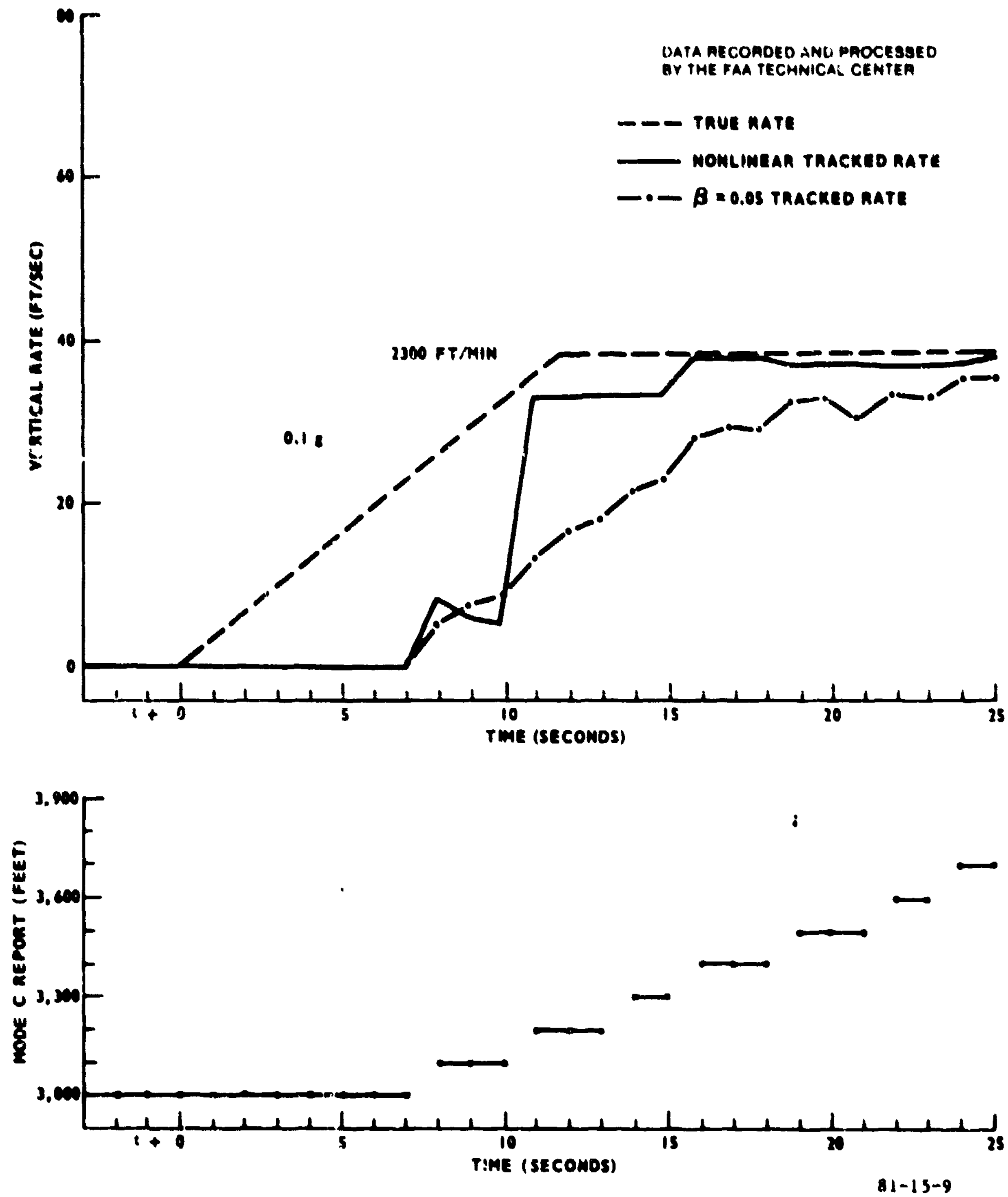

FIGURE 9. SLOW ACCELERATION AND NONCONSTANT BIN OCCUPANCY PERFORMANCE COMPARISON

The slow acceleration and final rate, which did not reflect constant bin occupancy times, resulted in the longest delays for the nonlinear tracker to provide accurate rate estimates. However, the nonlinear rate estimates were within 10 percent of the true rate for times greater than t+15. The same condition did not result for the $\alpha - \beta$ tracker until t+25.

Comparison of performance during high accelerations (g = 0.5) was also made. Figure 10 presents the results when the final rate is 2,300 ft/min. With the high acceleration, the final rate is obtained in 2.38 seconds. The first mode C transition occurs at t+4. Although the rate is only 38.33 ft/sec, a second transition occurs at t+6. Remember that the 2,300-ft/min rate reflects a combination of 3- and 2-second bin occupancy times. Because the second transition occurs only 2 seconds after the first, the nonlinear rate estimate exceeds the true rate by 5 ft/sec between t+6 and t+8. The magnitude of the error is less than one-half the error in the $\alpha - \beta$ rate estimate for the same period. After the third transition which occurs at t+9, the errors in the nonlinear rate estimate remain less than 2.62 ft/sec. Thus, for times greater than t+9, the peak error in the nonlinear tracker is less than 7 percent of the actual rate. This same accuracy does not occur in the $\alpha - \beta$ rate estimate until t+22. Again, the nonlinear tracker provided accurate rate estimates 12 seconds sooner than the $\alpha - \beta$ tracker.

The preceding analysis reviewed tracker performance for level flight aircraft which accelerate into constant rate climbs. Similar results can be expected for transitions from level flight into constant rate descents.

The comparison of the performance of the nonlinear and $\alpha - \beta$ trackers during accelerations from level flight is summarized in table 3. If the acceleration is initiated at time t:

$$\bar{E}_i = \frac{1}{10} \sum_{k=t+i-10}^{t+i} |R(K)|$$ where R(K) is the rate error at time k.

For instance, $\bar{E}_{10}$ is the average magnitude of the tracked rate error during the first 10 seconds following the acceleration. Similarly, $\bar{E}_{20}$ is the average magnitude of the tracked rate error during the period from 10 to 20 seconds after the initiation of the acceleration. The possible average magnitudes of the vertical position projection error can be obtained directly by multiplying table entries by 35 seconds. The comparisons are made for three different acceleration rates and final vertical rates ranging from 500 to 3,500 ft/min.

With the high acceleration rate (g = 0.5) there is essentially no difference in performance during the first 10 seconds following the acceleration when the final rate obtained is 1,500 ft/min or less. However, for final rates of 2,000 ft/min and above, the nonlinear average rate error approximates one-half to two-thirds of the $\alpha - \beta$ tracker's average rate error during the first 10 seconds following the acceleration.

During the 10- to 20-second time period following the acceleration, the nonlinear rate errors have been essentially eliminated regardless of the final rate obtained. The average rate errors for the $\alpha - \beta$ tracker remained significant during the t+10 to t+20 second time period when final rates obtained were 1,500 ft/min or more. The accuracy of $\alpha - \beta$ rate estimates does not match the uniform accuracy of the nonlinear rate estimates until t+20 seconds.

With 0.25 g acceleration rate, the same pattern of results is observed. For the t+10 to t+20 second time period, a larger difference in performance results than was observed with 0.5 g accelerations. Average $\alpha - \beta$ rate

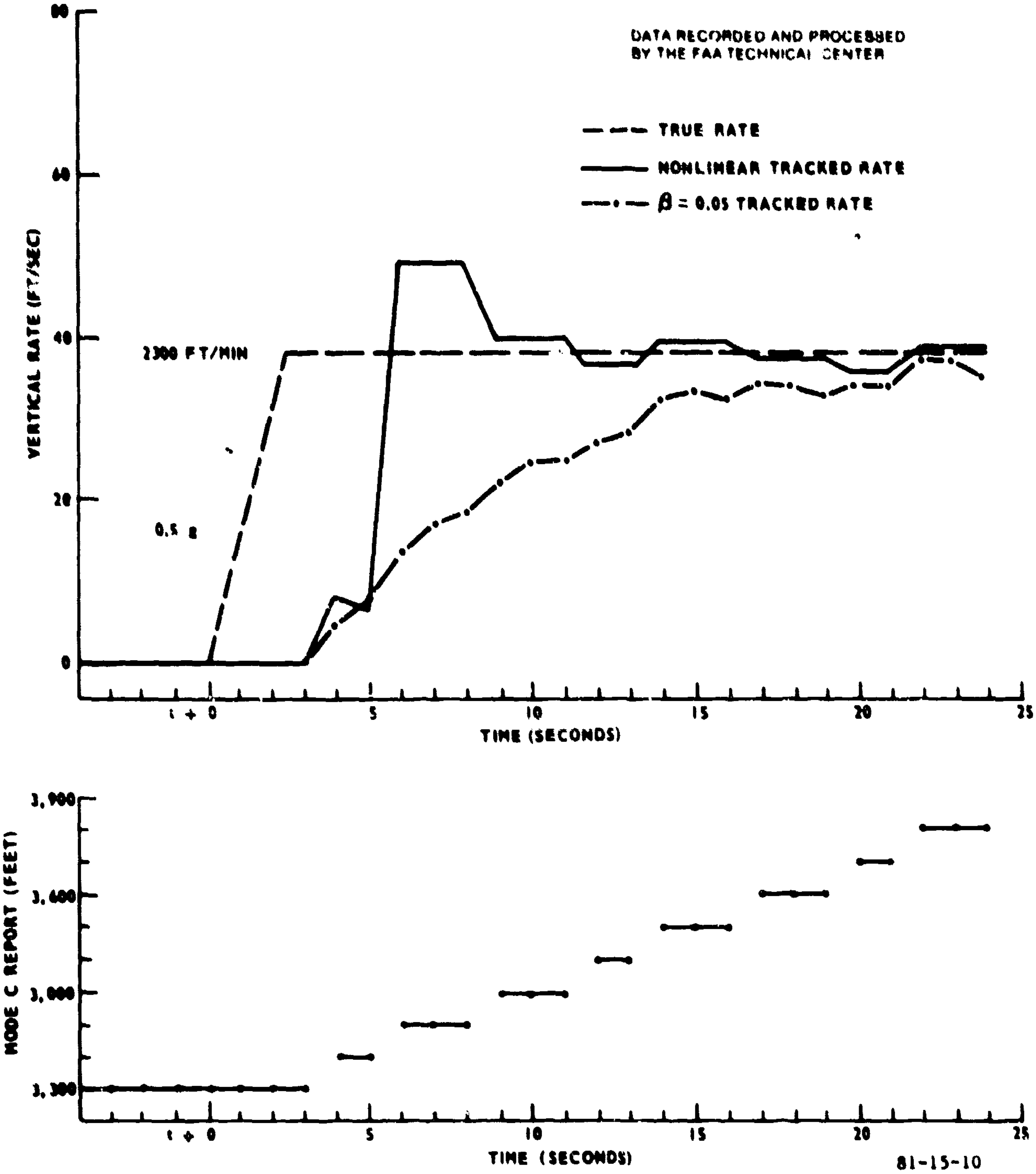

FIGURE 10. HIGH ACCELERATION AND NONCONSTANT BIN OCCUPANCY PERFORMANCE COMPARISON

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TABLE 3. COMPARISON OF TIME DEPENDENT AVERAGE ERROR MAGNITUDES DURING ACCELERATIONS — $\bar{E}_i$ FT/SEC

s	Final Rate (ft/min)	Nonlinear Tracker $\bar{E}_{10}$	Nonlinear Tracker $\bar{E}_{20}$	Nonlinear Tracker $\bar{E}_{30}$	$\alpha - \beta$ Tracker $\bar{E}_{10}$	$\alpha - \beta$ Tracker $\bar{E}_{20}$	$\alpha - \beta$ Tracker $\bar{E}_{30}$
0.50							
	500	8.33	2.86	1.65	8.33	2.31	1.58
	1,000	13.86	2.34	0.00	13.51	3.53	1.29
	1,500	16.41	0.00	0.00	18.03	4.43	0.99
	2,000	16.06	0.00	0.00	22.26	5.04	1.02
	2,500	18.64	1.43	0.61	27.26	6.18	1.40
	3,000	18.12	0.00	0.00	32.34	8.45	1.28
	3,500	18.09	1.13	0.77	35.14	9.48	1.39
0.25							
	500	9.08	3.43	1.66	9.08	2.94	2.64
	1,000	13.75	2.34	0.00	13.38	3.52	1.46
	1,500	15.32	0.00	0.00	17.71	4.35	0.90
	2,000	13.66	0.00	0.00	21.70	6.06	1.02
	2,500	18.09	1.48	0.67	25.09	7.46	1.72
	3,000	15.99	0.00	0.00	28.31	10.16	1.5[illegible]
	3,500	18.42	1.58	0.65	30.87	13.19	1.86
0.10							
	500	7.63	3.78	2.15	7.63	2.[illegible]3	2.59
	1,000	11.64	4.43	0.00	11.88	4.89	0.99
	1,500	14.34	1.92	0.00	14.46	7.27	1.54
	2,000	15.52	0.00	0.00	15.62	10.38	[illegible].56
	2,500	15.52	4.80	0.95	15.51	15.19	3.0[illegible]
	3,000	15.52	8.17	1.31	15.51	18.76	4.92
	3,500	15.52	10.34	1.30	15.51	22.16	7.62

errors for the t+10 to t+20 second time period could cause vertical projection errors to exceed 450 feet. As in the high acceleration case, the $\alpha - \beta$ tracker rate accuracy does not approach the nonlinear tracker accuracy until t+20 seconds.

For slow accelerations (g = 0.1), the performance of the two trackers is nearly identical during the first 10 seconds following the acceleration. This result occurs independently of the final rate obtained. When the final rate obtained exceeded 1,000 ft/min, the nonlinear tracker performed considerably better than the $\alpha - \beta$ tracker during the 10- to 20-second period following the acceleration. For slow accelerations, the $\alpha - \beta$ tracker rate errors persist during the t+20 to t+30 second time period.

With $\alpha - \beta$ tracking, projection errors as large as 265 feet could still persist more than 20 seconds after the acceleration.

DECELERATIONS TO LEVEL FLIGHT CONDITIONS.

In this section, we will review tracker performance when an aircraft in a fixed rate climb returns to level flight. The climb rates investigated ranged from 500 to 3,500 ft/min. The analysis of the tracker performance during the return to level flight from a climb rate of 2,300 ft/min will be presented. This rate represented the severest conditions analyzed.

In figure 11, a -0.1 g deceleration effect is presented. At -0.1 g, it takes 11.92 seconds to return to level flight from a 2,300-ft/min climb rate. The mode C transition pattern of 2- and 3-second bin occupancy times has been established in the climb. The detection of the deceleration requires the tracker to detect a deviation from the 2- and 3-second bin occupancy pattern. At t+6, the nonlinear tracker detects the excessive bin occupancy time following the deceleration. As a result, the rate estimate is adjusted with a decaying process. The same procedure is used at t+9. The adjustment almost eliminates the error in the nonlinear rate estimate. The final mode C transition occurs at t+6. At t+14, the nonlinear tracker observes that the estimated bin occupancy time, ZMOD7, has been exceeded by 5 seconds (the value of parameter P5). As a result, the rate estimate is set to 0 ft/sec to represent level flight. The $\alpha - \beta$ rate error exceeds 5.71 ft/sec for an additional 6 seconds. During this period, the $\alpha - \beta$ errors would result in at least a 200-foot error in vertical position projection.

For a higher deceleration rate (figure 12), the performance of the nonlinear tracker is even better when compared to the $\alpha - \beta$ tracker. For -0.5 g deceleration, an aircraft climbing at 2,300 ft/min can level off in 2.38 seconds. The last mode C transition, prior to deceleration, occurs at t-1. Hence, at t+3 the nonlinear tracker detects an excessive bin occupancy time and adjusts the rate estimate. At t+6, the estimated bin occupancy time has been exceeded by 5 seconds and the rate estimate is reset to 0 ft/sec. The $\alpha - \beta$ rate error does not decay below 5.71 ft/sec until t+12. Again, we see a 6-second difference in the establishment of accurate rate estimates.

During decelerations to level flight, the nonlinear tracker resets the rate estimate to 0 ft/sec when the excess bin occupancy time exceeds 5 seconds. The $\alpha - \beta$ tracker cannot reset its rate estimate to 0 ft/sec. The β value of 0.05 causes the decay in the rate estimate to 0 ft/sec to be quite slow. Depending on the precision of the logic arithmetic operations, the $\alpha - \beta$ rate estimate may cycle indefinitely with a nonzero magnitude. This could cause problems with the CAS logic's vertical speed limit routines. These routines make explicit checks for nonzero vertical rates.

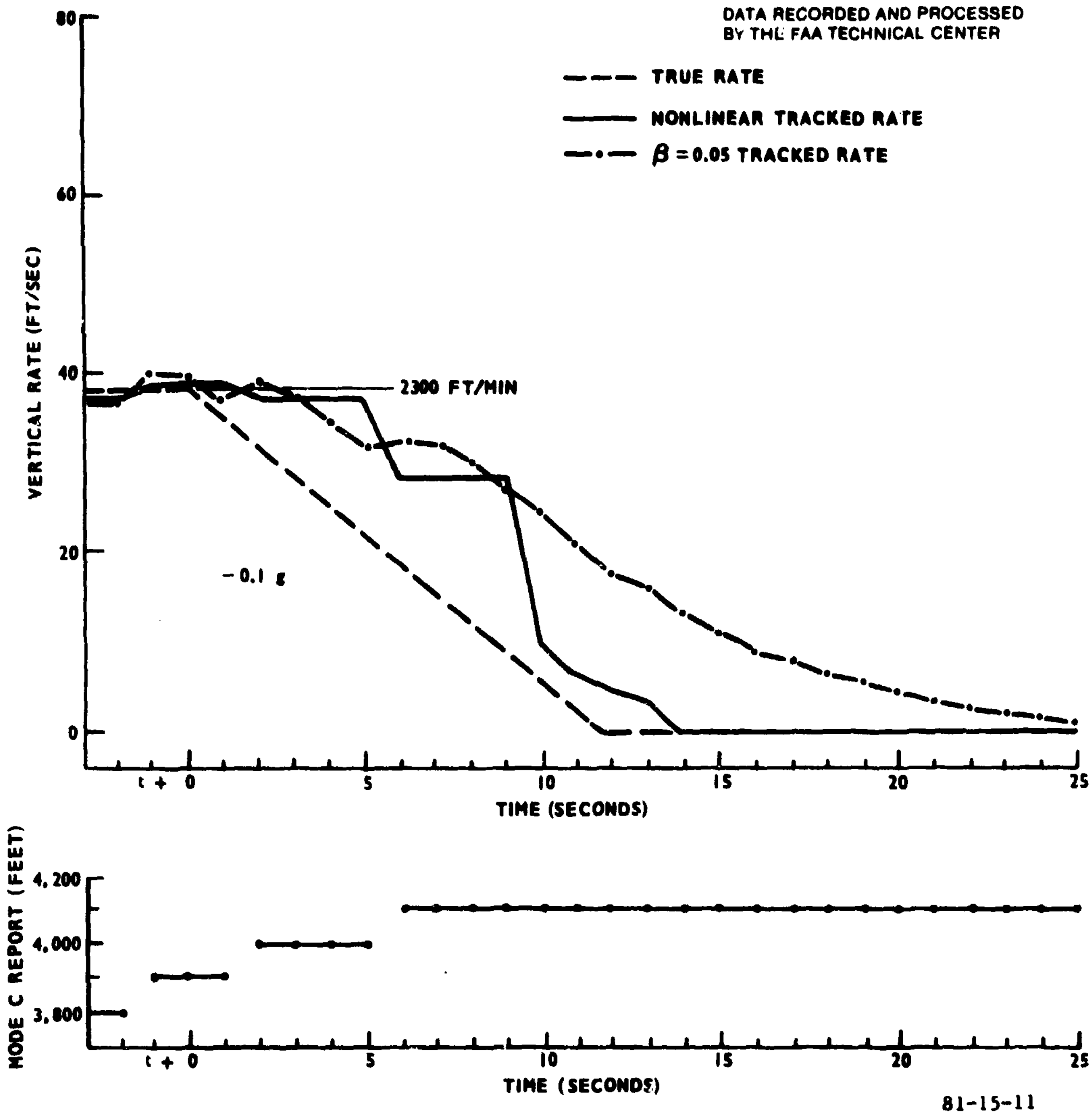

FIGURE 11. DECELERATION TO LEVEL FLIGHT CONDITION PERFORMANCE g = -0.1

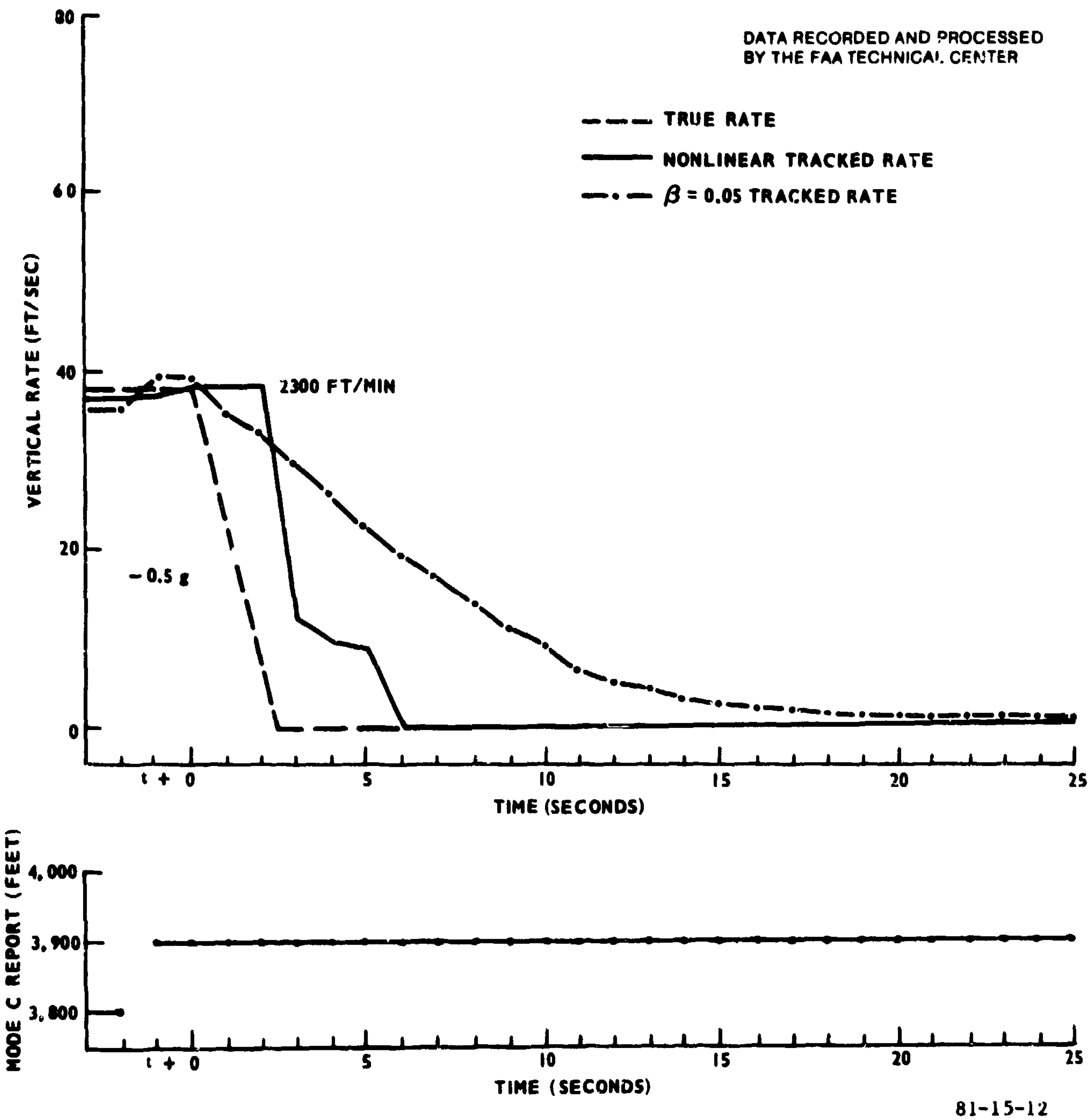

FIGURE 11. DECELERATION TO LEVEL FLIGHT CONDITION PERFORMANCE g = -0.5

Table 4 summarizes performance of the two trackers during and following decelerations from established climbs to level flight. The comparison is made for three deceleration rates and for established climb rates ranging from 500 to 3,500 ft/min. For almost all conditions tested, the nonlinear tracker declared level flight within 10 seconds of the initiation of the decelerations. The large errors that still exist in the nonlinear rate estimate more than 10 seconds after initiation of the deceleration occurred for the high initial climb rate (3,000 and 3,500 ft/min) and for the slow deceleration rate (-0.1 g). In these cases, level flight conditions do not occur until 15.5 and 18.1 seconds after the deceleration was initiated. The $\alpha - \beta$ rate estimates contained a considerably larger error than the nonlinear tracker during the 10- to 20-second time period following the initiation of deceleration.

DECELERATION TO OTHER THAN LEVEL FLIGHT CONDITIONS.

Analysis of tracker performance during decelerations from a fixed rate to another rate (not level flight) was made. In figure 13, the initial rate is -2,500 ft/min and at time t a deceleration to -1,000 ft/min is initiated. With a -0.25 g deceleration, the new rate is obtained in 3.10 seconds. At t+3, the nonlinear tracker detects an excess in the bin occupancy time and adjusts the rate estimate. Between t+3 and t+5, the nonlinear tracker overestimates the deceleration. However, the magnitude of the rate error of the nonlinear track is less than the $\alpha-\beta$ tracker error magnitude. Beyond t+6, the nonlinear tracker rate estimate is error free. Consistently accurate $\alpha - \beta$ rate estimates do not occur until t+14, an 8-second difference in the delay.

PITCHOVER MANEUVERS.

Maneuvers in which a constant rate deceleration takes an aircraft from a constant climb rate into a constant descent rate were analyzed. In figure 14, the aircraft is established in a 3,000-ft/min climb rate, then decelerates into a 1,000-ft/min descent. The deceleration rate presented is -0.5 g. The nonlinear tracker detects excess bin occupancy time at t+3 and adjusts the rate estimate to approximately 1,000 ft/min. At t+7, a downward transition in altitude bins is detected and a negative rate estimate occurs. The $\alpha - \beta$ tracker does not provide a negative rate estimate until t+13. At this time, the nonlinear rate estimate is error free. In general, the analysis of pitchover maneuver results indicated significantly earlier (6 to 10 seconds) negative rate detection by the nonlinear tracker. The differences in time, to obtain accurate final rate estimates, approximated differences identified in the comparison of deceleration performance.

During pitchover maneuvers the ability of the vertical tracker to determine when the direction of vertical movement has changed is critical. The parameter ZDLVL is used to discount low vertical rates by intruder aircraft in selecting maneuver sense. The delay in the tracker detecting a rate less than ZDLVL is a good measure of tracker performance during pitchover maneuvers. Tracker responses during and following -0.5 g pitchover maneuvers were analyzed. Initial climb rates varied between 500 and 3,500 ft/min. The -0.5 g pitchover was sustained until a 1,000-ft/min descent rate was established.

Table 5 identifies the results of the pitchover analysis. The comparison is made in terms of the delay in seconds until certain tracked conditions exist. Since the delay periods are dependent on the location within an altitude bin when the pitchover maneuver is initiated, the delay is stated as a period of time. Since the maximum rate estimates for both trackers are less than 10 ft/sec for a 500-ft/min climb, the tracked

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TABLE 4. COMPARISON OF TIME-DEPENDENT AVERAGE ERROR MAGNITUDE DURING DECELERATIONS TO LEVEL FLIGHT

g	Initial Climb Rate (ft/min)	Nonlinear Tracker $\bar{E}_{10}$	Nonlinear Tracker $\bar{E}_{20}$	Nonlinear Tracker $\bar{E}_{30}$	$\alpha-\beta$ Tracker $\bar{E}_{10}$	$\alpha-\beta$ Tracker $\bar{E}_{20}$	$\alpha-\beta$ Tracker $\bar{E}_{30}$
-0.50							
	500	6.76	0.0	0.0	4.10	0.64	0.08
	1,000	13.67	0.0	0.0	13.84	2.89	0.45
	1,500	7.41	0.0	0.0	12.65	2.34	0.30
	2,000	12.51	0.0	0.0	19.87	4.11	0.46
	2,500	5.52	0.0	0.0	20.86	4.36	0.58
	3,000	9.40	0.0	0.0	25.31	5.78	0.76
	3,500	18.74	0.0	0.0	32.41	8.83	1.20
-0.25							
	500	6.46	0.0	0.0	4.07	0.70	0.08
	1,000	12.08	0.0	0.0	12.98	3.25	0.45
	1,500	5.63	0.0	0.0	10.76	2.34	0.30
	2,000	8.50	0.0	0.0	16.45	4.21	0.55
	2,500	12.72	0.0	0.0	22.39	7.03	0.96
	3,000	16.16	0.0	0.0	23.90	8.93	1.25
	3,500	20.10	0.0	0.0	28.07	12.12	1.72
-0.10							
	500	5.79	0.0	0.0	3.40	0.74	0.07
	1,000	10.68	0.48	0.0	10.39	3.25	0.45
	1,500	11.24	0.61	0.0	11.24	5.49	0.80
	2,000	11.71	0.70	0.0	11.71	7.92	0.98
	2,500	13.62	0.51	0.0	13.62	10.32	2.06
	3,000	10.83	6.71	0.0	10.83	16.08	5.00
	3,500	10.71	9.90	0.18	10.71	19.81	6.94

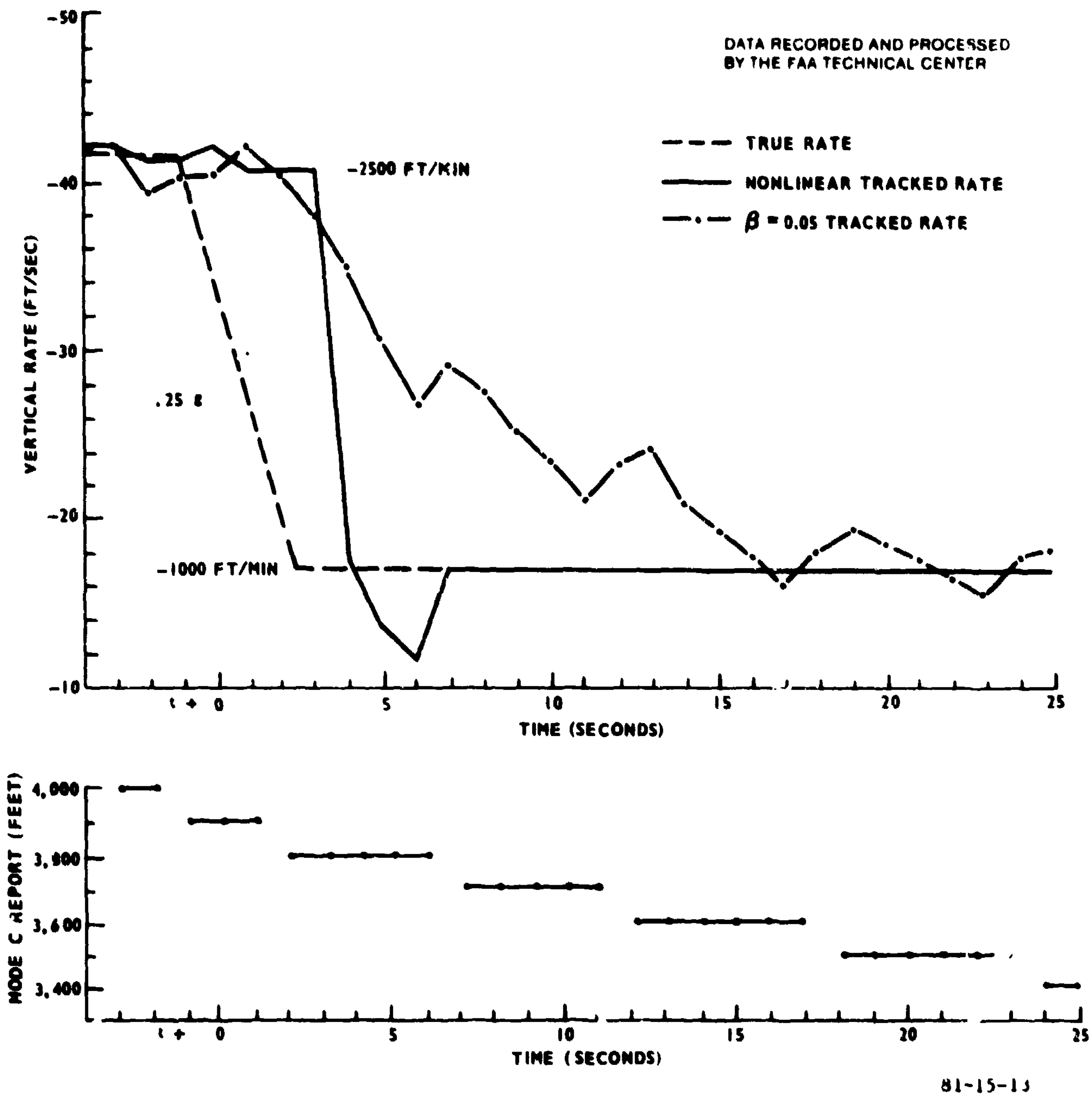

FIGURE 13. COMPARISON DURING DECELERATION TO FIXED RATE OTHER THAN LEVEL FLIGHT

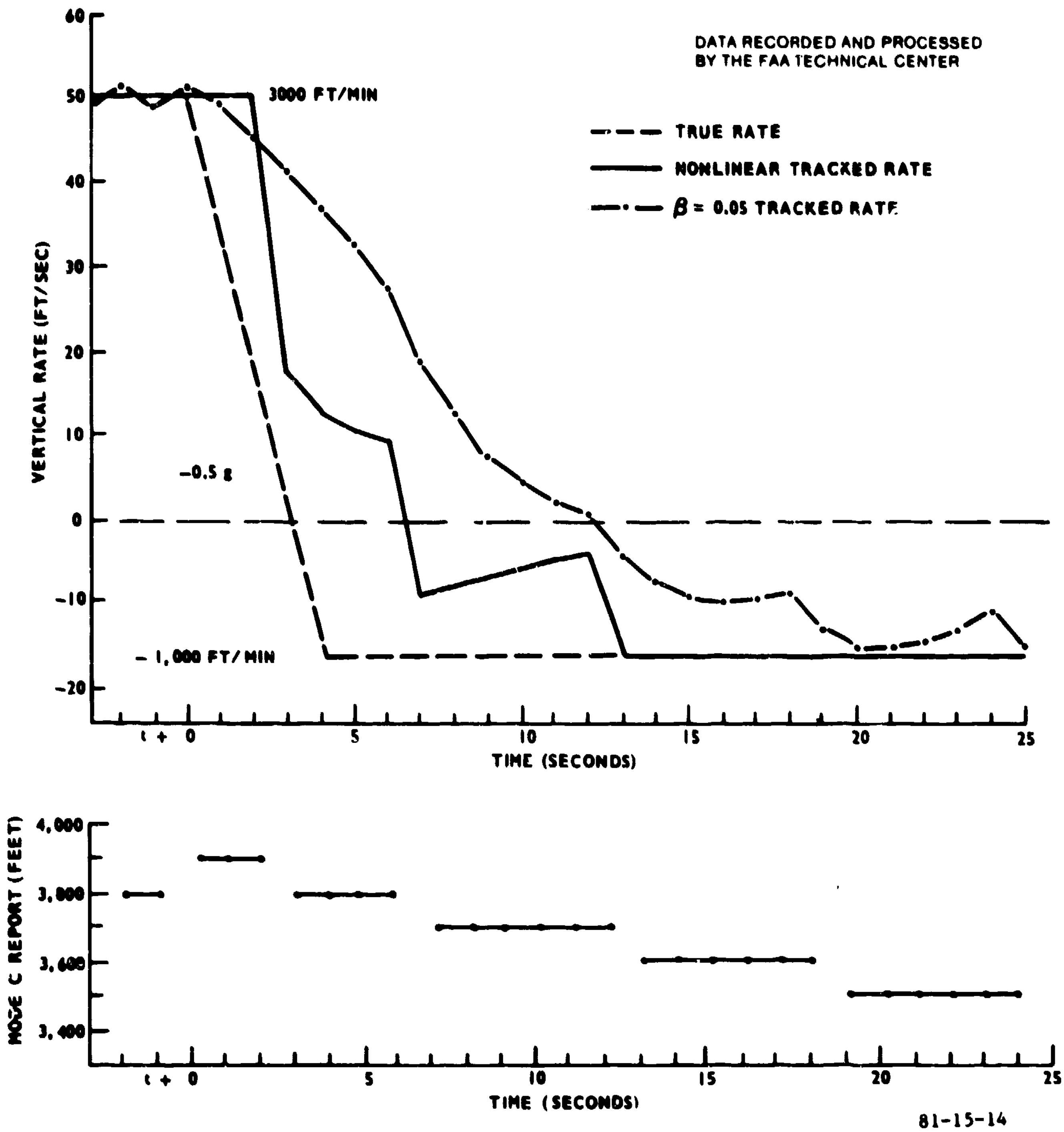

FIGURE 14. PITCHOVER MANEUVER PERFORMANCE COMPARISON

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TABLE 5. COMPARISON OF TRACKER RESPONSE DELAYS FOLLOWING
-0.5 g PITCHOVER MANEUVER TO -1,000 FT/MIN

Initial Climb Rate (ft/min)	Period Tracked Rate Exceeds 10 ft/sec (seconds) Nonlinear	$\alpha-\beta$	Period Until Descent is Detected (seconds) Nonlinear	$\alpha-\beta$
500	-	-	4-6	5-7
1,000	2-7	5-9	5-8	8-13
[illegible]	3-6	6-10	6-8	8-13
2,000	3-6	7-10	4-7	10-14
2,500	4-7	8-11	6-8	12-14
2,700	4-7	8-11	6-8	12-14
3,000	4-8	9-11	6-8	13-15
3,500	5-8	10-12	6-8	13-16

rates do not exceed 10 ft/sec during the pitchover when the initial climb rate is 500 ft/min. The maximum time required to depress the nonlinear rate estimate below 10 ft/sec following the initiation of the pitchover is, in general, less than the minimum delay that is expected with $\alpha - \beta$ tracking. The delay with $\alpha - \beta$ tracking required before a descent is detected is twice the delay experienced with nonlinear tracking. This result is fairly uniform across all initial climb rates above 1,000 ft/min.

STAIRSTEP MANEUVER.

The final stand-alone tracker comparisons analyzed stairstep vertical profiles. The aircraft was assumed to be descending at a constant rate. A level-off maneuver was then accomplished, followed by deceleration into a descent 5 to 20 seconds later. Figure 15 presents a typical vertical profile.

The results are shown in figure 16. The major advantage of the nonlinear tracker is the ability to obtain an accurate level flight rate estimate much sooner than the $\alpha - \beta$ tracker. The nonlinear tracker provides the level flight rate estimates for the time period between t+10 and t+25. The $\alpha - \beta$ tracker obtains accurate level flight rate estimates only for times greater than t+20. Once the descent is resumed at t+26, the nonlinear tracker responds as if the aircraft had always been in level flight. As long as the level flight period exceeds the altitude strata occupancy time (3 seconds for a 2,000-ft/min vertical rate) by 5 seconds, the nonlinear tracker will reset the vertical rate estimate to 0 ft/min for the level flight portion of the stairstep maneuver.

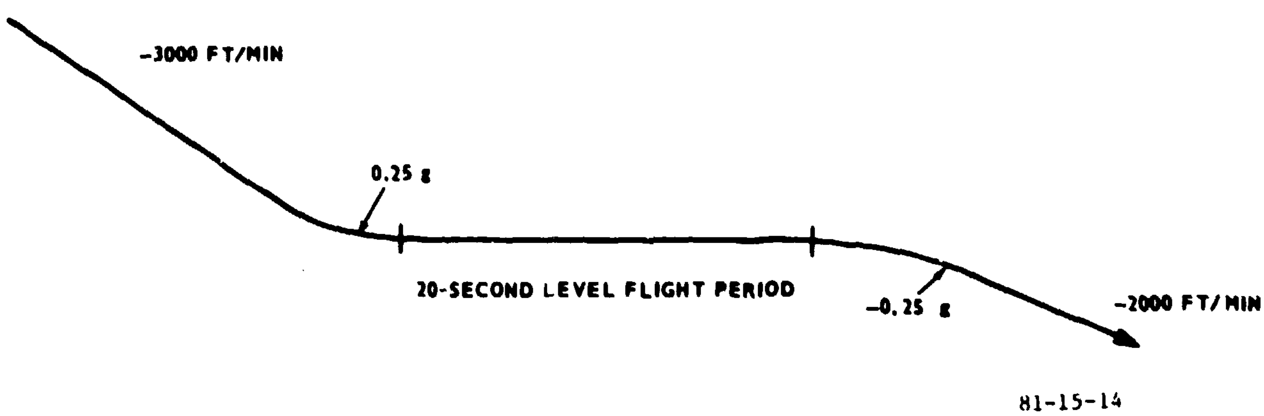

FIGURE 15. TYPICAL STAIRSTEP MANEUVER

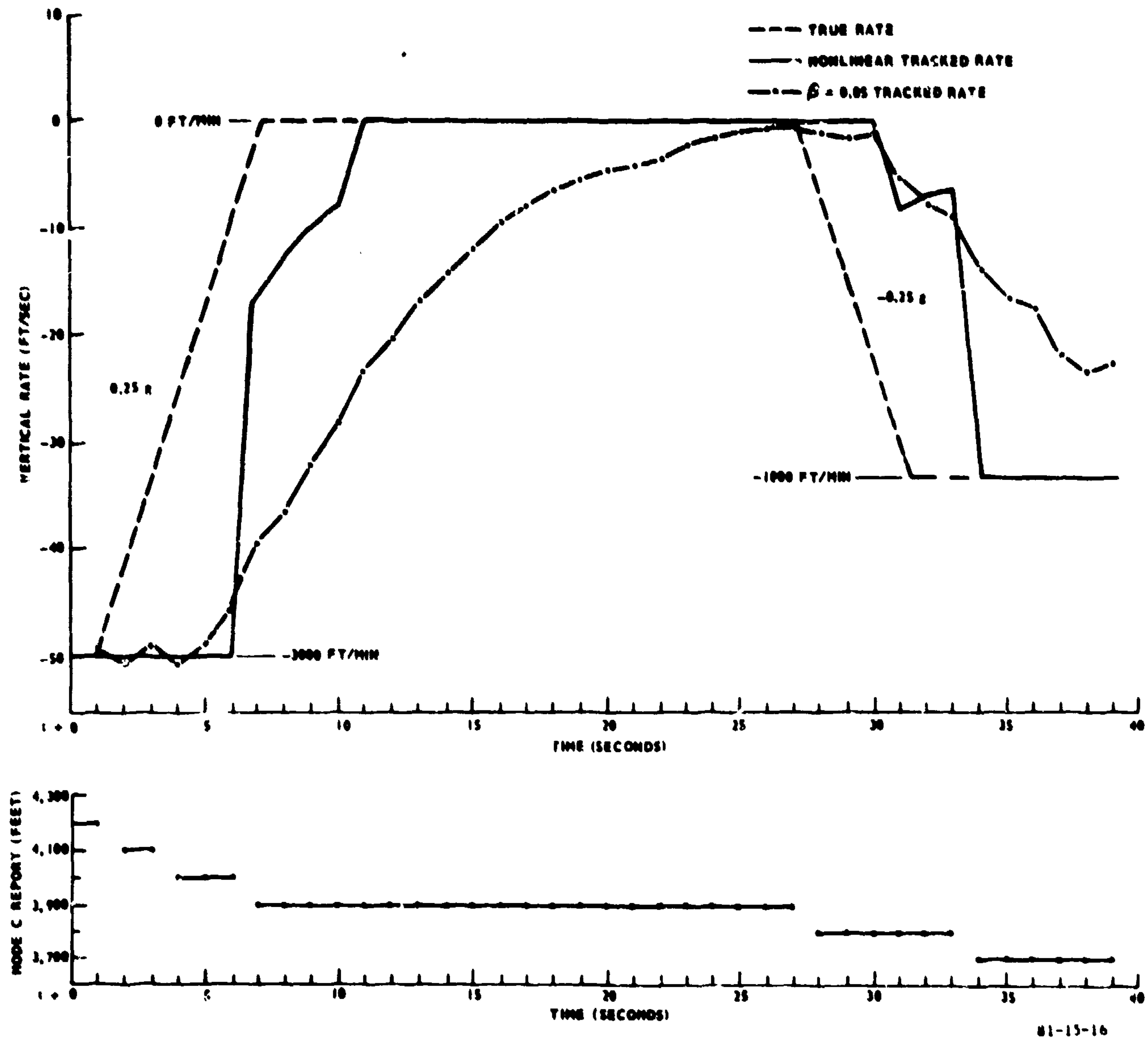

FIGURE 16. STAIRSTEP MANEUVER PERFORMANCE COMPARISON

FAST-TIME ENCOUNTER GENERATOR RESULTS.

Previous problems with command sense choice during intruder accelerations were identified in reference 8. Unequipped intruder encounter scenarios like the sequence depicted in figure 17 resulted in BCAS commands which reduced vertical separation at CPA. Fast-time simulation analysis was conducted to see if the earlier detection of accelerations by the nonlinear tracker had improved the collision avoidance logic performance for scenarios similar to those shown in figure 17.

The variety of geometries analyzed are shown in figure 18. The Active BCAS logic does not permit sense of command to change once it is selected. If the wrong sense is selected and the logic selects a positive command, vertical separation at CPA is reduced by the incorrect sense choice. The response model used for this analysis included a 5-second pilot response delay.

The question that is addressed in this section is "How long does the error in the vertical estimate, which would cause the wrong command sense to be selected, persist following the level off maneuver by the intruder?" For conditions shown in figure 17, the incorrect sense choice for the BCAS aircraft is climb. Since sense choice can occur on any logic cycle when all threat detection conditions are satisfied, the improvement in sense choice because of nonlinear vertical tracking can be identified by comparing the period of incorrect sense choice associated with each tracker following the level off maneuver.

Table 6 presents the reduction in duration of the incorrect sense choice period which occurred with nonlinear tracking. Following the level-off maneuver, both the $\alpha - \beta$ and nonlinear tracking rate estimates contain a residual error. For a specific planned separation value, an incorrect sense choice results when the residual rate error projected over 35 seconds exceeds the planned vertical separation value. The improvement in sense choice performance with nonlinear tracking is due to a faster tracker response to the variation in the mode C report pattern.

Table 6 indicates that sense choice improvement increases with the magnitude of the initial vertical rate. For larger values of vertical separation, larger residual errors are necessary for incorrect sense choices to result. For lower initial vertical rates (less than -1400 ft/min) and the larger planned vertical separations, the duration of incorrect sense choice periods for both trackers are nearly identical. However, for higher initial vertical rates, nonlinear tracking reduced the incorrect sense periods by up to 7 seconds.

Table 7 presents the improvement with nonlinear tracking that results when the level-off maneuver is performed more rapidly (-0.5 g versus -0.25 g).

Although a considerable reduction in the duration of the incorrect sense choice period occurs with nonlinear tracking, wrong sense choices may still occur. Wrong sense choices result with the nonlinear tracker because the sense selection logic does not make optimum use of the nonlinear tracker. Research should be conducted to improve the use of nonlinear tracking data by the sense choice logic. Two immediate considerations should be investigated. Since command sense cannot change once it is selected, when sufficient time (-range/range rate sufficiently large) until CPA remains, command sense selection should be delayed during periods of acceleration by the intruder. Several internal variables in the nonlinear tracker (ZMOD8 or a function of ZMOD7) appear to be good indicators of acceleration by the intruder. A second, more sophisticated approach is to base sense determination on vertical position projection which includes a vertical acceleration factor.

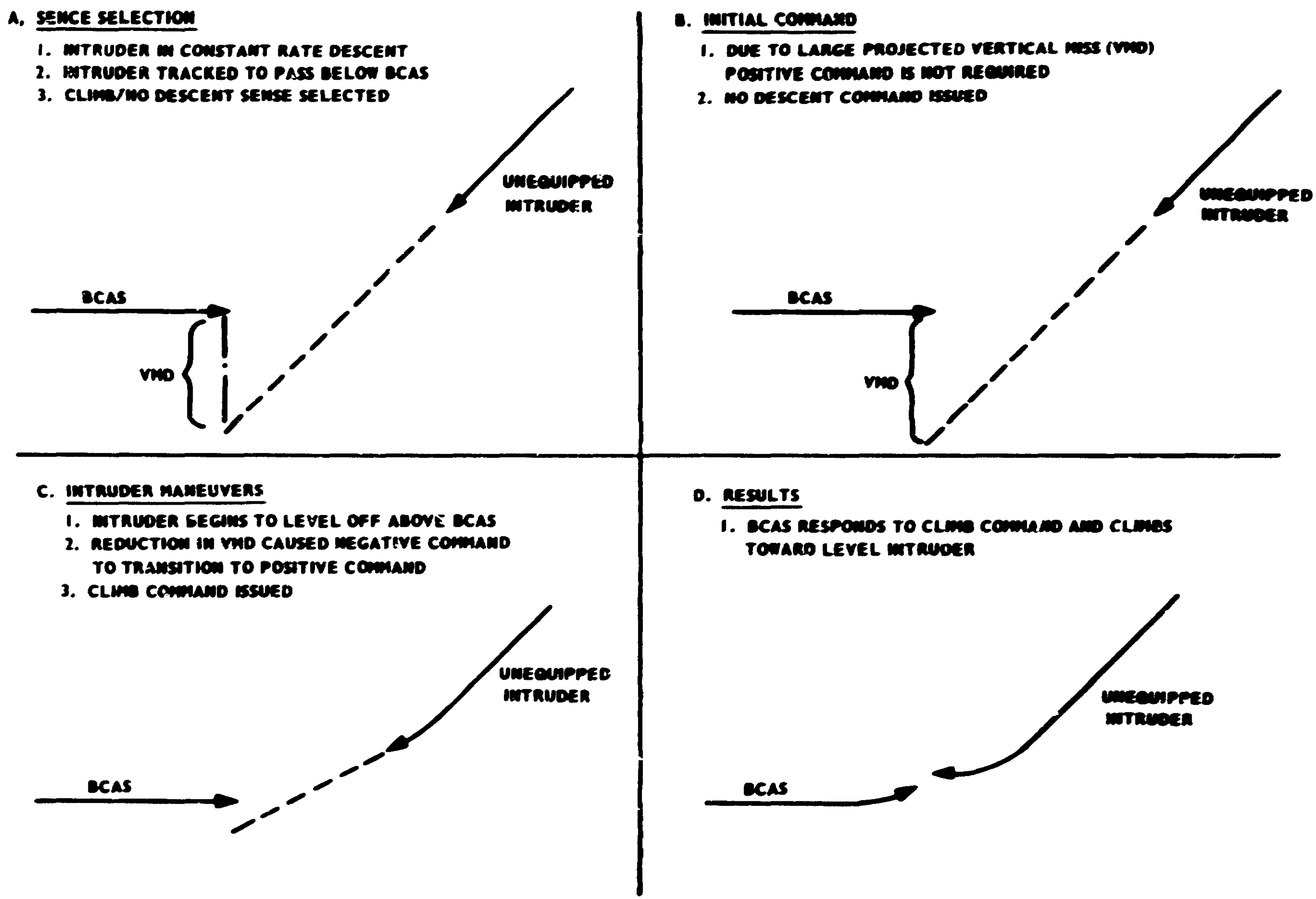

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FIGURE 17. UNEQUIPPED INTRUDER ENCOUNTER SEQUENCE WHICH RESULTS IN INCORRECT SENSE CHOICE

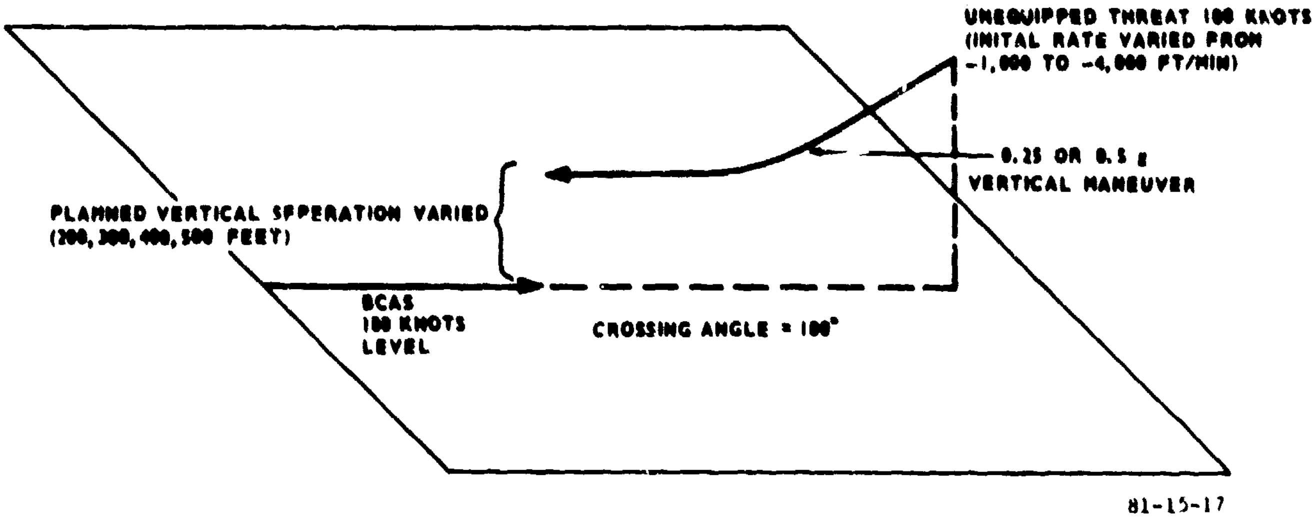

FIGURE 18. GEOMETRIES USED IN COMPARISON OF DETECTION PERFORMANCE

REVIEW OF LIVE FLIGHT TEST RESULTS.

During Active BCAS flight testing, an encounter occurred with a DC-9 aircraft. The Federal Aviation Administration (FAA) BCAS aircraft, registration number N49, was level at 16,000 feet. The mode C report history revealed that the DC-9 was climbing at approximately 2,700 ft/min. About 35 seconds prior to the tracked minimum range time, the mode C report history showed a vertical deceleration by the DC-9. The highest mode C report observed was 15,500 feet. The deceleration rate was about (-0.5 g). The deceleration period lasted approximately 4 seconds. The mode C report history shows a final vertical rate of about -1,000 ft/min.

The vertical profile for this encounter is shown in figure 19. The initial BCAS alarm, do not climb, occurs 4 seconds after the mode C history shows a deceleration by the DC-9. At this time, the projected vertical separation, Zp, is calculated as follows:

Zp = BCAS altitude - (DC-9 tracked position + 35 seconds x DC-9 tracked altitude rate)

For the α - β tracker in the airborne experimental BCAS unit

$$Zp = 16,000 - (15,543 + 35*(34)) = -593 \text{ ft}$$

This implies the DC-9 is projected to be 593 feet above the level flight BCAS aircraft at CPA. As a result, the BCAS logic displayed a "do not climb" command for the BCAS aircraft.

This encounter was simulated using the fast-time encounter generator. The time

TABLE 6. REDUCTION IN DURATION OF INCORRECT SENSE CHOICE PERIOD IN SECONDS ACHIEVED WITH NONLINEAR VERTICAL TRACKING (-0.25 g DECELERATION)

Initial Vertical RATE (Ft/Min)	Planned Vertical Separation at CPA Following Level-Off Maneuver (Feet) 200	300	400	500
-1000	2	2	0	0
-1100	2	2	1	0
-1200	3	2	1	0
-1300	4	3	2	0
-1400	4	4	3	2
-1500	4	4	3	3
-1600	4	4	3	3
-1700	5	4	3	3
-1800	5	5	4	3
-1900	5	5	4	4
-2000	5	5	5	4
-2100	5	5	4	4
-2200	5	5	5	4
-2300	6	6	5	4
-2400	6	6	5	4
-2500	6	6	5	5
-2600	6	6	5	4
-2700	6	5	6	5
-2800	5	5	5	5
-2900	6	6	5	4
-3000	6	6	5	4
-3100	7	6	6	6
-3200	7	7	6	6
-3300	6	6	5	4
-3400	7	5	5	6
-3500	6	7	6	5
-3600	6	5	4	5
-3700	6	6	6	6
-3800	5	4	3	3
-3900	7	6	6	5
-4000	7	8	7	6

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TABLE 7. REDUCTION IN DURATION OF INCORRECT SENSE CHOICE PERIODS IN SECONDS ACHIEVED WITH NONLINEAR VERTICAL TRACKING (-0.5 g DECELERATION)

Initial Vertical RATE (Ft/Min)	Planned Vertical Separation at CPA Following Level-Off Maneuver (Feet) 200	300	400	500
-1000	2	2	0	0
-1100	3	3	2	0
-1200	3	2	2	0
-1300	3	3	2	1
-1400	4	4	3	2
-1500	4	4	3	2
-1600	4	4	3	2
-1700	4	4	3	2
-1800	5	5	4	4
-1900	5	5	4	4
-2000	5	5	4	4
-2100	6	5	5	4
-2200	6	5	5	4
-2300	6	5	5	5
-2400	6	6	6	5
-2500	7	7	6	5
-2600	7	7	7	6
-2700	7	7	7	7
-2800	6	6	6	5
-2900	7	7	7	6
-3000	7	7	7	7
-3100	7	7	7	6
-3200	8	8	7	6
-3300	7	7	7	7
-3400	8	8	7	7
-3500	7	7	7	7
-3600	8	8	8	8
-3700	7	7	7	7
-3800	7	7	7	7
-3900	8	7	7	7
-4000	8	7	7	7

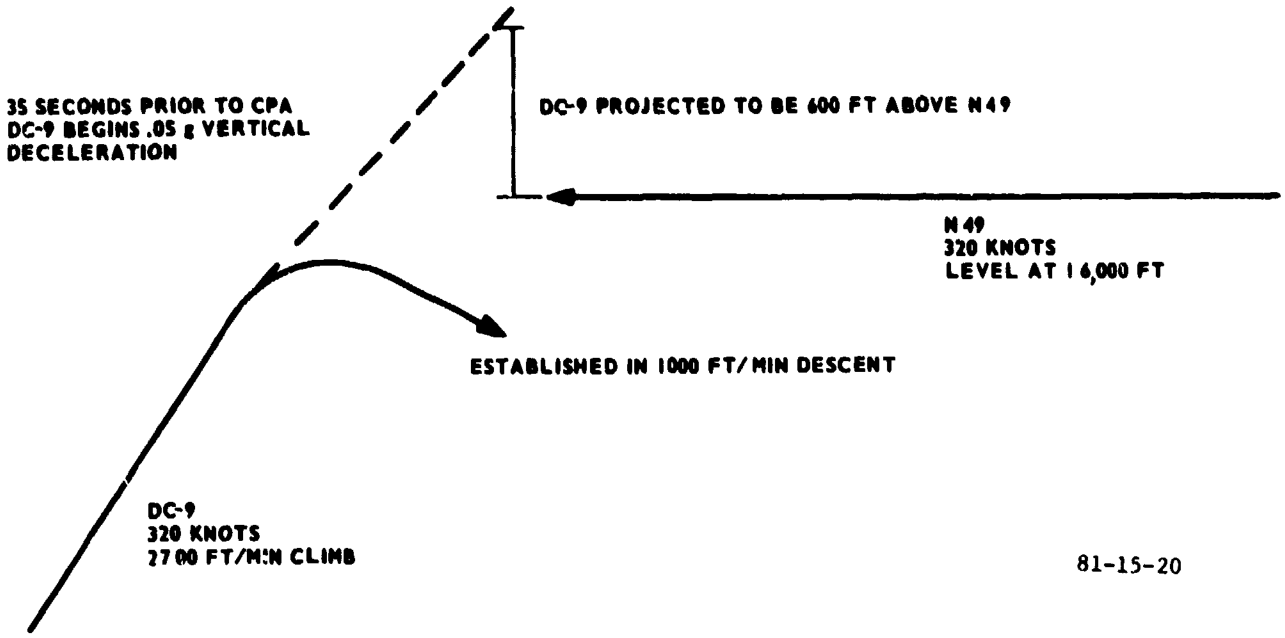
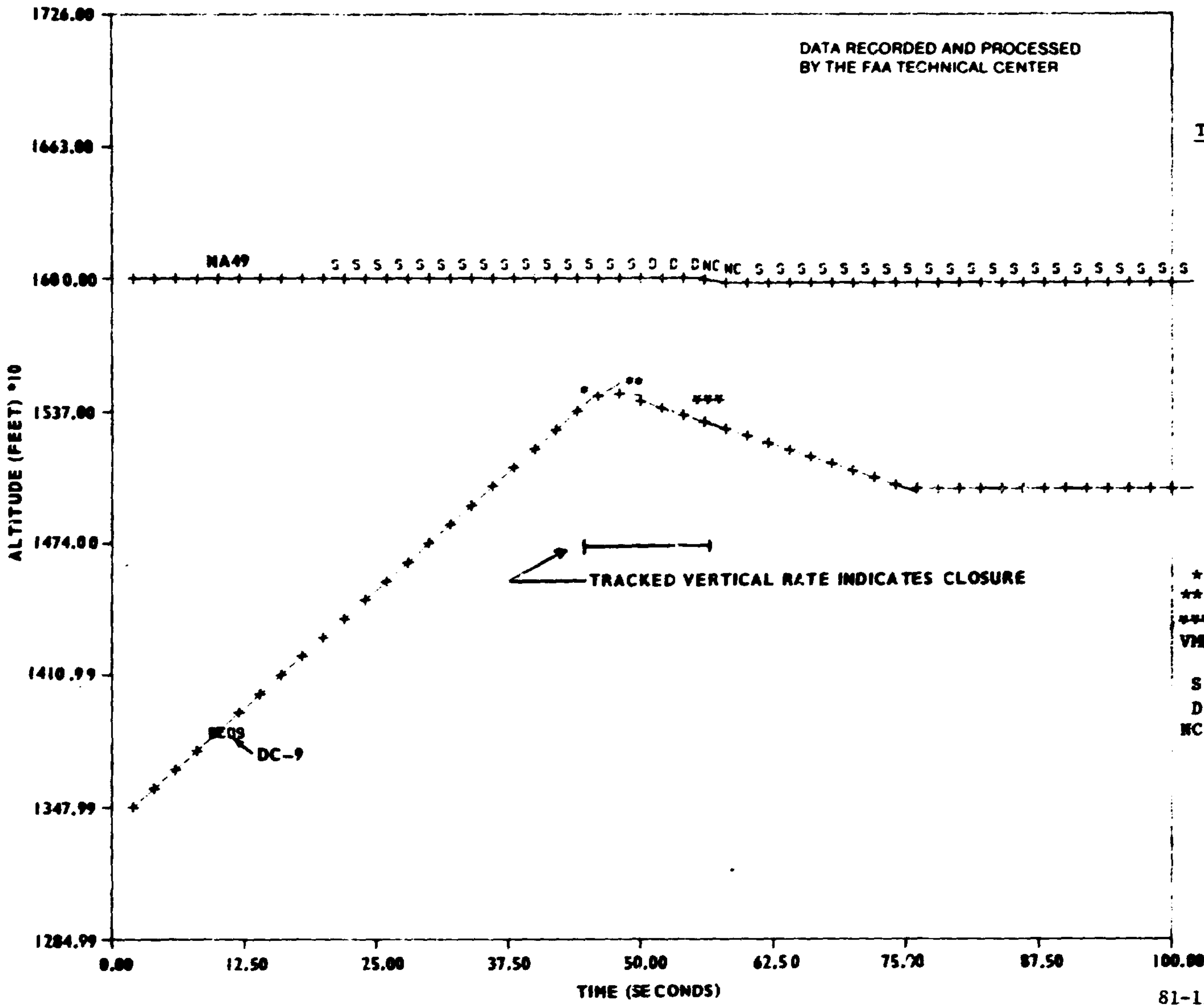


FIGURE 19. VERTICAL PROFILE OF DC-9 ENCOUNTER

versus altitude plot of the results which occurred with $\alpha - \beta$ tracking is shown in figure 20. In simulation, the sense of the initial alarm, which was descend, was incorrect. In simulation, command selection appears to have occurred 1 second later than during the actual flight. If a first hit had occurred at the 48th second, the command selected would have been "do not climb" since the absolute value of VMD $|-544| > 470$ feet, the threshold for positive commands. The logic requires a command to be displayed for a minimum of 5 seconds prior to being replaced by another command. A review of the live flight data indicated a positive descent command did not result because the projected vertical separation was less than 470 feet during the 5-second period immediately following the display of the initial "do not climb" alarm. The $\alpha - \beta$ tracker rate estimates for the DC-9 remained positive for 12 seconds following the vertical deceleration. During this period, the collision avoidance logic would sense the positive rate as vertical closure and possibly select an incorrect command sense.

By repeating the fast-time simulation, results were obtained for nonlinear tracking. Figure 21 shows the results. With the nonlinear tracker, a negative vertical rate estimate occurs 5 seconds after the deceleration. Since sense selection occurs at this time, the sense selected climb is correct. Since the projected vertical miss distance of 670 feet exceeds the positive command threshold of 470 feet, a "do not descend" command is displayed.

The tracker responses associated with this encounter were investigated further by supplying the DC-9 mode C history directly to the trackers independent of the collision avoidance logic. The tracker responses are shown in figure 22. Assuming the deceleration was initiated at time t, the duration of the periods of incorrect sense choice



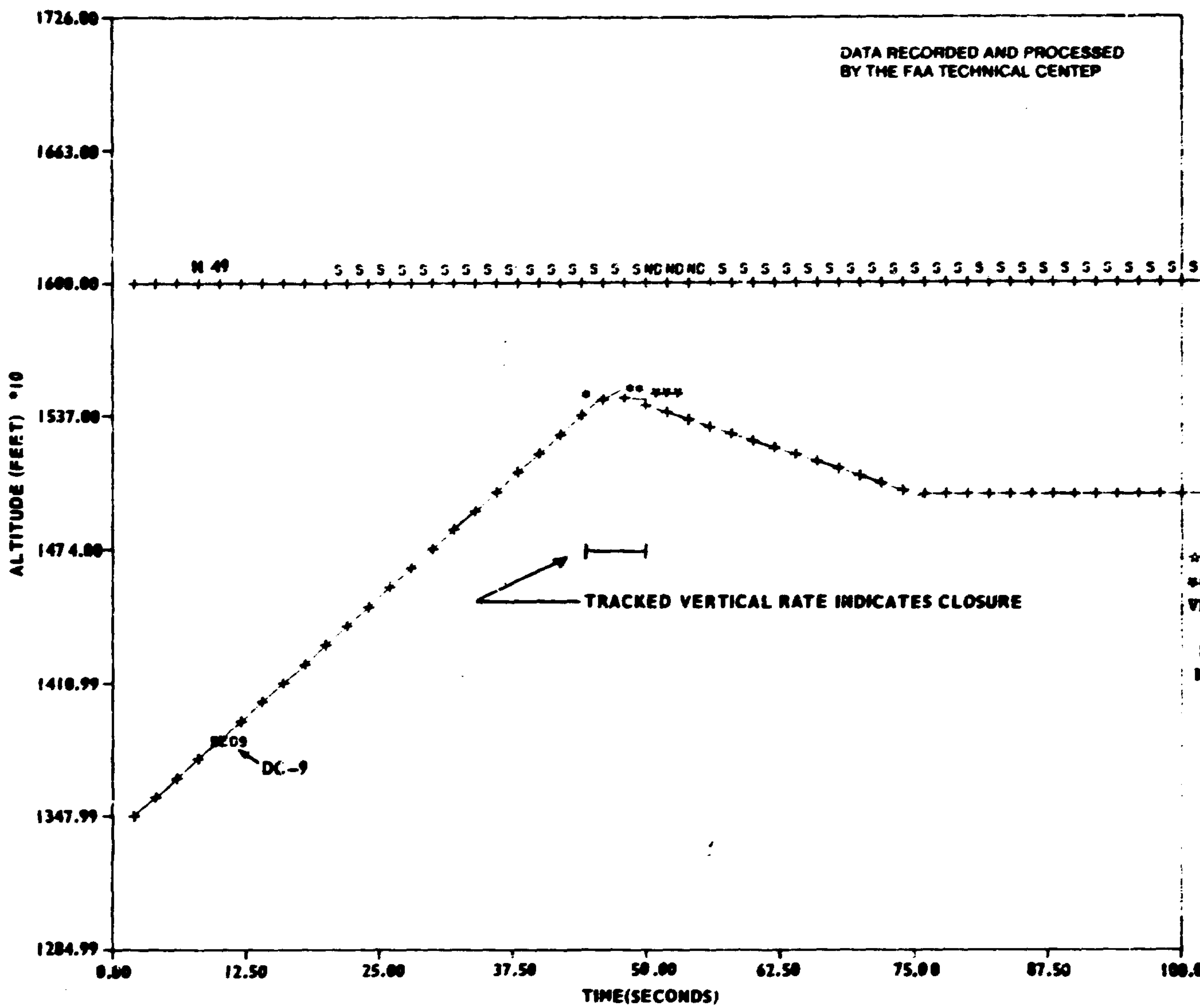
ACT - 220/CSC

TIME	VMD	COMMAND
44	-	-
45*	-980	-
46	-864	-
47	-913	-
48	-855	-
49**	-544	1ST HIT
50	-299	DESCEND
51	-104	"
52	48	"
53	168	"
54	263	"
55	503	NO CLIMB
56***	647	"
57	704	"
58	737	"
59	753	"
60	760	NONE

* RATE CHANGE OCCURRED
** RATE CHANGE DETECTED
*** TRACKED RATE IS NEGATIVE
VMD PROJECTED VERTICAL SEPARATION (FEET)
S PROXIMITY WARNING
D DESCEND
NC NO CLIMB

81-15-21

FIGURE 20. LIVE FLIGHT TEST ENCOUNTER — ALPHA-BETA TRACKING RESULTS



ACT - 220/CSC

TIME	VMD	COMMAND
44	-	-
45*	-1007	-
46	-987	-
47	-969	-
48**	-958	-
49***	662	1ST HIT
50	670	NO DESCENT
51	673	"
52	673	"
53	671	"
54	667	"
55	883	"
56	901	NONE

* RATE CHANGE OCCURRED
** RATE CHANGE DETECTED
*** TRACKED RATE IS NEGATIVE
VMD PROJECTED VERTICAL SEPARATION (FEET)
S PROXIMITY WARNING
ND NO DESCENT

81-15-22

FIGURE 21. LIVE FLIGHT TEST ENCOUNTER — NONLINER TRACKING RESULTS

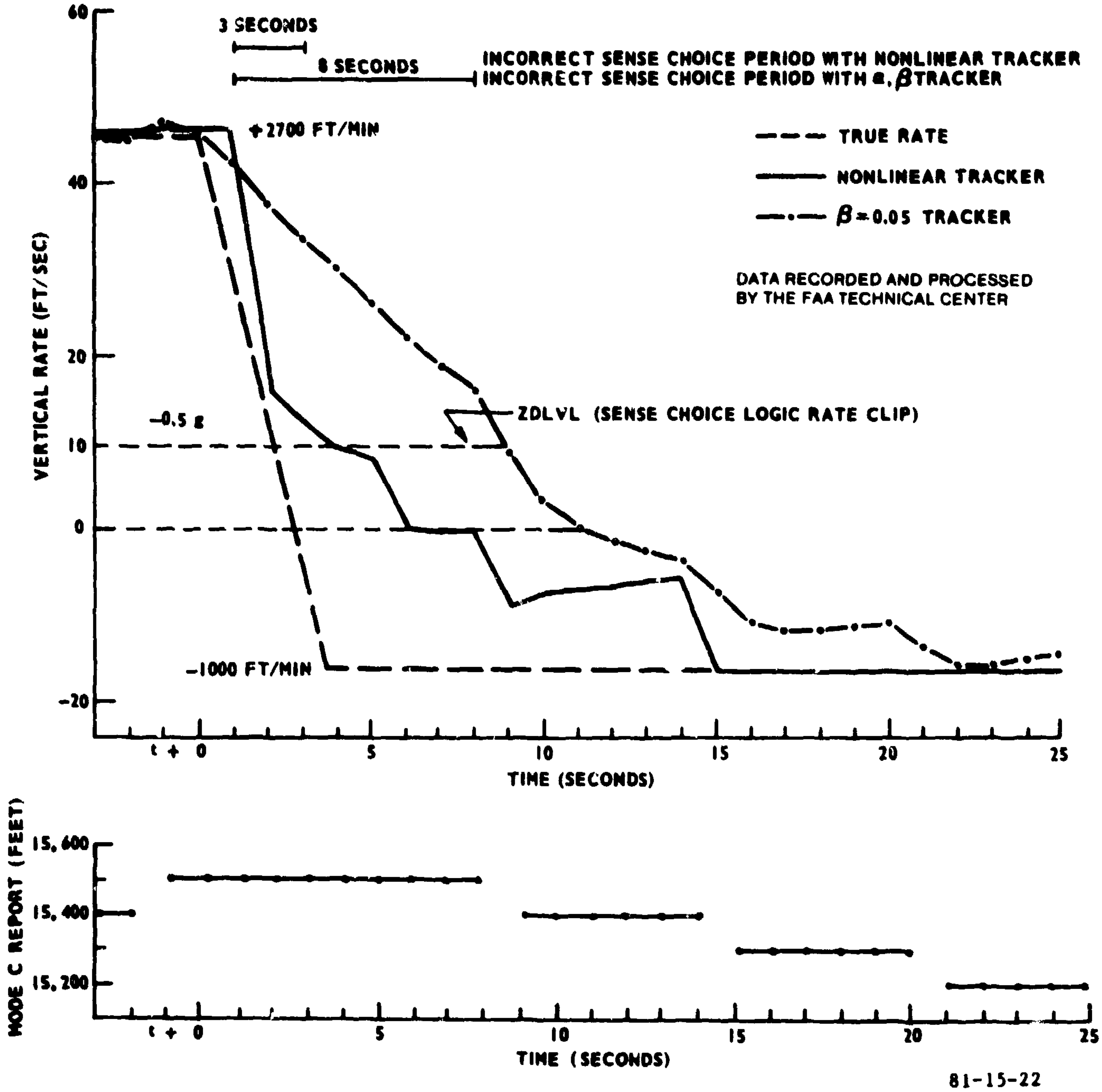


FIGURE 22. TRACKER RESPONSES FOR DC-9 VERTICAL PROFILE

for each tracker were obtained. The sense choice logic has a vertical rate threshold, ZDLVL, such that sense choice is based strictly on current relative position when the ablolute value of the rate estimate is less than ZDLVL (10 ft/sec). For the DC-9 vertical profile, the duration of the incorrect sense choice period equals the time it takes the tracker to depress the rate estimate below 10 ft/sec. With the nonlinear tracker, this occurs at t+4; hence, the incorrect sense choice period is 3 seconds. The $\alpha - \beta$ tracker does not depress the rate estimate below 10 ft/sec until t+9, resulting in an 8-second incorrect sense choice period. This corresponds closely with the live flight test results. The rate estimate in the flight test results remained above 10 ft/sec for 10 seconds following the DC-9 deceleration.

With the nonlinear tracker, the stand-alone analysis indicated a level flight condition was declared 6 seconds after the deceleration was initiated. The $\alpha - \beta$ tracker required 12 seconds to establish the same result. This difference indicates the $\alpha - \beta$ tracker would declare a vertical closure condition for 6 more seconds than the nonlinear tracker.

CONCLUSIONS

1. Review of $\alpha - \beta$ tracker performance through the use of fast-time simulation results in the following conclusions:

a. The previous rate tracking constant value of $\beta = 0.15$ had been developed for use with 4.7-second data rates. The much faster data rate (1 second) of Active Beacon Collision Avoidance System (BCAS) caused large errors to be induced into the rate estimate by mode C quantization. The large errors often resulted in incorrect command sense choice even during steady state vertical maneuvers by the threat aircraft.

b. A reduction in β to 0.1 somewhat reduced the occurrence of incorrect sense choice during steady state vertical maneuvers. However, initial flight testing showed that 100-foot mode C excursions for aircraft in level flight could result in rate errors exceeding 16 feet per second (ft/sec).

c. Modifications to the sense choice logic and a reduction in β to 0.05 eliminated sense choice problems associated with 100-foot mode C report excursions. However, the decrease in β has resulted in a significant increase in the tracker's transient rate response delay.

2. Review of the nonlinear tracker described in the appendix led to the following conclusions:

a. The complexity increase when compared to the $\alpha - \beta$ tracker requires the storage of ten additional logic parameters. The number of elements in the own aircraft state vector increased by eight. Also, the number of elements in each intruder threat file increased by eight.

b. The missing or garbled altitude report flag in the Collision Avoidance System (CAS) logic ZFLG was easily incorporated into the nonlinear tracking logic to indicate when missing altitude report logic should be used.

3. Comparative analysis of the performance of the nonlinear tracker with the $\alpha - \beta$ tracker resulted in the following conclusions:

a. For 100-foot mode C excursions, the maximal rate error associated with nonlinear tracking was 8 ft/sec. This occurs because transitions in the mode C reports for aircraft with tracked vertical rates of 0 ft/sec are treated as isolated transitions. The maximal error occurs at the time of transition and decays even when successive reports reinforce the transition. The maximal

error in the rate estimate is less than ZDLVL. Hence, incorrect sense choices do not occur during 100-foot mode C report excursions with nonlinear tracking.

b. During steady state conditions for all rates analyzed, the nonlinear tracker peak error in the rate estimate was significantly smaller than the $\alpha - \beta$ tracker peak error in the rate estimate.

c. During steady state conditions for vertical rates which represent constant bin occupancy times, the nonlinear tracker would provide error-free rate estimates. The rates associated with constant bin occupancy times (500, 1,000, 1,500, 2,000 feet per minute (ft/min)) are rates at which aircraft normally maneuver. Regardless of rate, the $\alpha - \beta$ tracking rate estimate cycles with a periodicity equal to the bin occupancy times.

d. Following acceleration from level flight into constant rate climbs, little difference in the times required for each tracker to obtain accurate rate estimates is detected for final vertical rates $\leq$1,000 ft/min. However, as the final rate is increased, nonlinear tracking results in accurate rate estimates significantly earlier (6 to 12 seconds) than with the $\alpha - \beta$ tracker. This fact appears uniform across the acceleration rates tested.

e. The weakest performance by the nonlinear tracker occurs during slow accelerations to final rates which are not associated with constant bin occupancy times. However, accurate rate estimates are obtained considerably earlier with the nonlinear tracker than with the $\alpha - \beta$ tracker.

f. During and following level-off maneuvers, the nonlinear tracker required significantly less time (6 to 10 seconds) to obtain 0 ft/sec rate estimates. When the excess bin occupancy time exceeds 5 seconds, the rate estimate is reset to 0 ft/sec. The $\alpha - \beta$ tracker's rate estimate can oscillate around 0 ft/sec indefinitely, depending on the arithmetic precision of the logic.

g. During pitchover maneuvers earlier detection of the change in the sign of the vertical rate occurs with the nonlinear tracker. The large transient response delay associated with $\beta = 0.05$ results in long periods in which the $\alpha - \beta$ tracking rate estimate has the wrong sign.

h. During stairstep maneuvers whenever the level flight duration exceeds 5 seconds plus the expected bin occupancy time, the nonlinear rate estimate is reset to the correct value of 0 ft/sec. This does not occur with $\alpha - \beta$ tracking.

i. Fast-time simulation of Active BCAS logic with both nonlinear and $\alpha - \beta$ tracking has shown that a significant improvement in command sense choice occurs with nonlinear tracking. Additional improvement in sense choice performance during periods of acceleration may be obtained. This improvement may be possible by augmenting the sense selection logic with logic that accounts for accelerating intruders.

4. Review of live flight test sense choice anomalies indicates that the occurrence of incorrect sense choice would be reduced with nonlinear tracking.

RECOMMENDATIONS

Based on the results of the analysis of the nonlinear tracker, it is recommended that the review tracker be incorporated into Active Beacon Collision Avoidance System (BCAS) flight testing. Furthermore, it is recommended that additional analysis be made to identify methods of optimizing Collision Avoidance System (CAS) logic sense selection. The optimization should make use of the additional information that is available with nonlinear tracking that was not previously available with Alpha-Beta tracking.

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APPENDIX A

NONLINEAR VERTICAL TRACKER

Only minor changes were made to the code provided by Lincoln Laboratory in the VTRK66 Version of the nonlinear altitude tracker. The changes included: (1) the parameter P2 was deleted since it was not referenced in the code, (2) the previous $\alpha - \beta$ tracking of own altitude was replaced with the nonlinear tracking logic, and (3) the check for missing or unreliable altitude reports (ZM=0) was changed to conform with the flag provided by Beacon Collision Avoidance System (BCAS) surveillance tracking which indicates missed, garbled, or an unreliable altitude report, ZFLAG=0. This change permitted the direct interface of VTRK66 into the Collision Avoidance System (CAS) logic intruder tracking routine, TRIACT.

To support the addition of VTRK66, specific parameters had to be added to the BCAS CAS logic parameter list. The parameters added are shown in table A-1.

Two other parameters used in VTRK66 logic were not needed by the CAS logic. The parameter DT, which identifies the nominal update rate (1 second) already exists within the CAS tracking modules. The BCAS variables TDATA and TDATAI already determine the duration of missing report periods. The quantization bin with parameter Q was not added to the parameter list. The constant 100 was added in its place in the VTRK66 logic. Two parameters were deleted from the BCAS parameter list. They were the $\alpha - \beta$ position tracking constant, ALFAZ, and the rate tracking constant, BETAZ.

VTRK66 logic required that several new elements be added to the intruder track file. The elements are shown in table A-2.

Each element in table A-2 is subscripted with the intruder's ID or track file number. The same elements were also added to the own aircraft state vector. It is important to note that ZMOD6 was added to the intruder state vector although the same information already exists in the state vector in the form of TDATA. This was done because ZMOD6 is internally manipulated in the VTRK66 logic and may have an effect on other uses of TDATA in the CAS logic.

Certain additional track file elements now require initialization prior to use by the tracker. When the BCAS is first made operational, own intruder state elements must be initialized. The additional elements requiring initialization are shown in table A-3.

When intruder reports are first passed to the CAS logic by the surveillance logic, procedures must initialize certain new elements in the intruder track file. The initialization is performed in TRIACT. Nonlinear tracking occurs on subsequent reports. The elements of the intruder state vector to be initialized are shown in table A-4.

When an intruder track file exists and surveillance tracking provides at least a range report on a particular logic cycle, the nonlinear tracking logic as shown in figure A-1 is used to provide intruder tracking. This logic will handle unreliable, missing, or garbled altitude reports. When no surveillance report is received for an intruder with an established track file, coasting of the track with the nonlinear tracker is accomplished as shown in figure A-2. Own tracking is accomplished by integrating the nonlinear tracker into TROACT in the same manner shown in figure A-1. The only difference is that the missing altitude report logic can be deleted.

TABLE A-1. BCAS CAS LOGIC PARAMETER LIST ADDITIONS

Parameter	Definition	Nominal Value
P1	Magnitude of rate allowed following isolated altitude transition	8 feet per second
P3	Rate decay factor when no reinforcing transition has occurred	0.90
P4	Stiff rate smoothing parameter	0.04
P5	Excess bin occupancy time which results in transition to level flight	5 seconds
P6	Excess bin occupancy time which results in correction to rate estimate	1.5 seconds
P7	Amount of discrepancy in bin occupancy time which triggers rate reinitialization	1.5 seconds
P8	Parameter used to position an estimated bin transition within an interval of missing reports	0.6
P9	Position smoothing parameter	0.3
P10	Decay factor for residual monitor, ZMOD10, for scans without detection of excess residual	0.8
P11	Bin occupancy smoothing parameter when excess residuals are detected	0.6
P12	Decay factor following a correction to ZMOD10	0.75
P13	Value of ZMOD9 at which transition from start up smoothing to normal smoothing occurs	18

TABLE A-2. ELEMENTS ADDED TO INTRUDER TRACK FILE

VTRK66 Element	Previous Track File Element	Definition	Units
ZM	ZRINT	Mode C report. ZFLAG=0 when report is missing or garbled	feet (100)
ZMOD1	ZINT	Tracked altitude of intruder	feet
ZMOD2	ZDINT	Tracked altitude rate of intruder	feet per second
ZMOD3	New Element	Time last mode C report was received for this intruder	seconds
ZMOD4	New Element	Previous mode C report for this intruder	feet (100)
ZMOD5	New Element	Time of transistion to previous mode C altitude for this intruder	seconds
ZMOD6	New Element	Time of last CAS track update for this intruder	seconds
ZMOD7	New Element	Estimated bin occupancy time	seconds
ZMOD8	New Element	Firmness of rate, typically equal to number of altitude transitions observed for current rate	--
ZMOD9	New Element	Start up counter needed to establish rate	--
ZMOD10	New Element	Residual indicator. Used to detect a trend in tracker residuals which indicate vertical acceleration	--

TABLE A-3. OWN STATE VECTOR INITIALIZATION PROCEDURE

VTRK66 Element	Previous CAS Logic Element	Initialization Value
ZMOD1	ZOWN	Current own mode C report
ZMOD2	ZDOWN	0 feet per second**
ZMOD3	New Element	TCUR*
ZMOD4	New Element	Current mode C report
ZMOD5	New Element	Added below
ZMOD6	New Element	TCUR
ZMOD7	New Element	100/(ABS(ZMOD2)+0.1)
ZMOD8	New Element	5
ZMOD9	New Element	0
ZMOD10	New Element	0
ZMOD5	New Element	TCUR-ZMOD7 + 7
ZM	ZROWN	Current mode C report

*Variable name for current BCAS time in CAS logic.

**If BCAS is turned on in flight there should be an initial standby period. This will permit the possible observance of a bin occupancy period. The standby period should last 5 seconds or until two mode C transitions are observed, whichever is shorter. If less than two transitions occurred, ZMOD2 should be initialized to 0 ft/sec. If two transitions are observed, then ZMOD2=(mode C report on second transition — mode C report on first transition)/time between transitions. This procedure will reduce the time delay required to obtain accurate own aircraft rate estimates when own aircraft is maneuvering at high vertical rates (above 20 ft/sec) when BCAS is first initialized.

TABLE A-4. INTRUDER STATE VECTOR INITIALIZATION PROCEDURES

VRTK66 Element	Previous CAS Track File Element	Initialization Value
ZMOD1	ZINT	Surveillance altitude report ZRINT
ZMOD2	ZDINT	Surveillance rate ZDSV
ZMOD3	New Element	TCUR
ZMOD4	New Element	ZRINT
ZMOD5	New Element	Added below
ZMOD6	New Element	TCUR
ZMOD7	New Element	100/(ABS(ZDSV)+0.1)
ZMOD8	New Element	5
ZMOD9	New Element	0
ZMOD10	New Element	0
ZMOD5	New Element	TCUR-ZMOD7 + 7
ZM	ZRINT	ZRINT

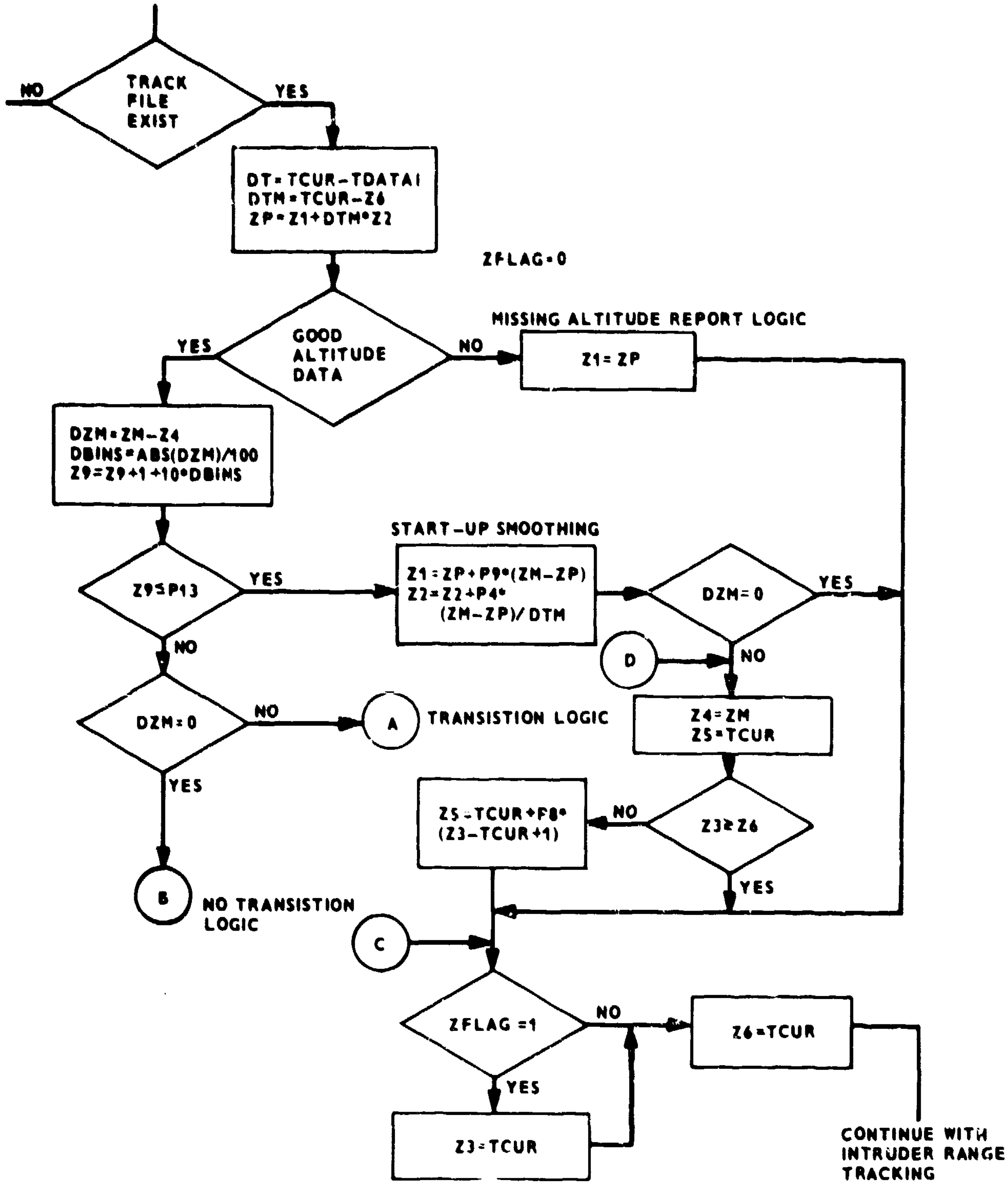

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FIGURE A-1. NONLINEAR TRACKING LOGIC AS INTEGRATED INTO TRIACT (Sheet 1 of 3)

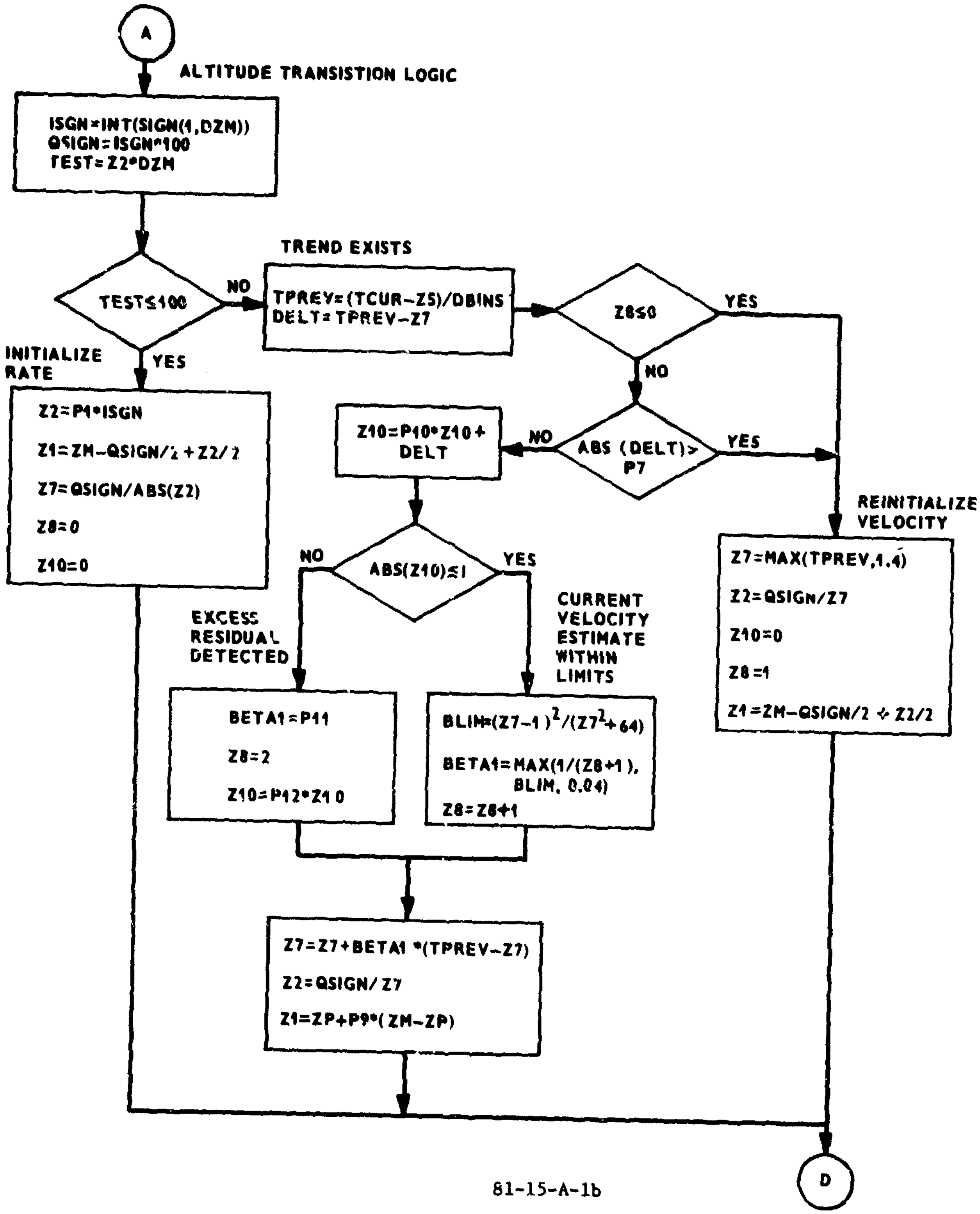

FIGURE A-1. NONLINEAR TRACKING LOGIC AS INTEGRATED INTO TRIACT (Sheet 2 of 3)

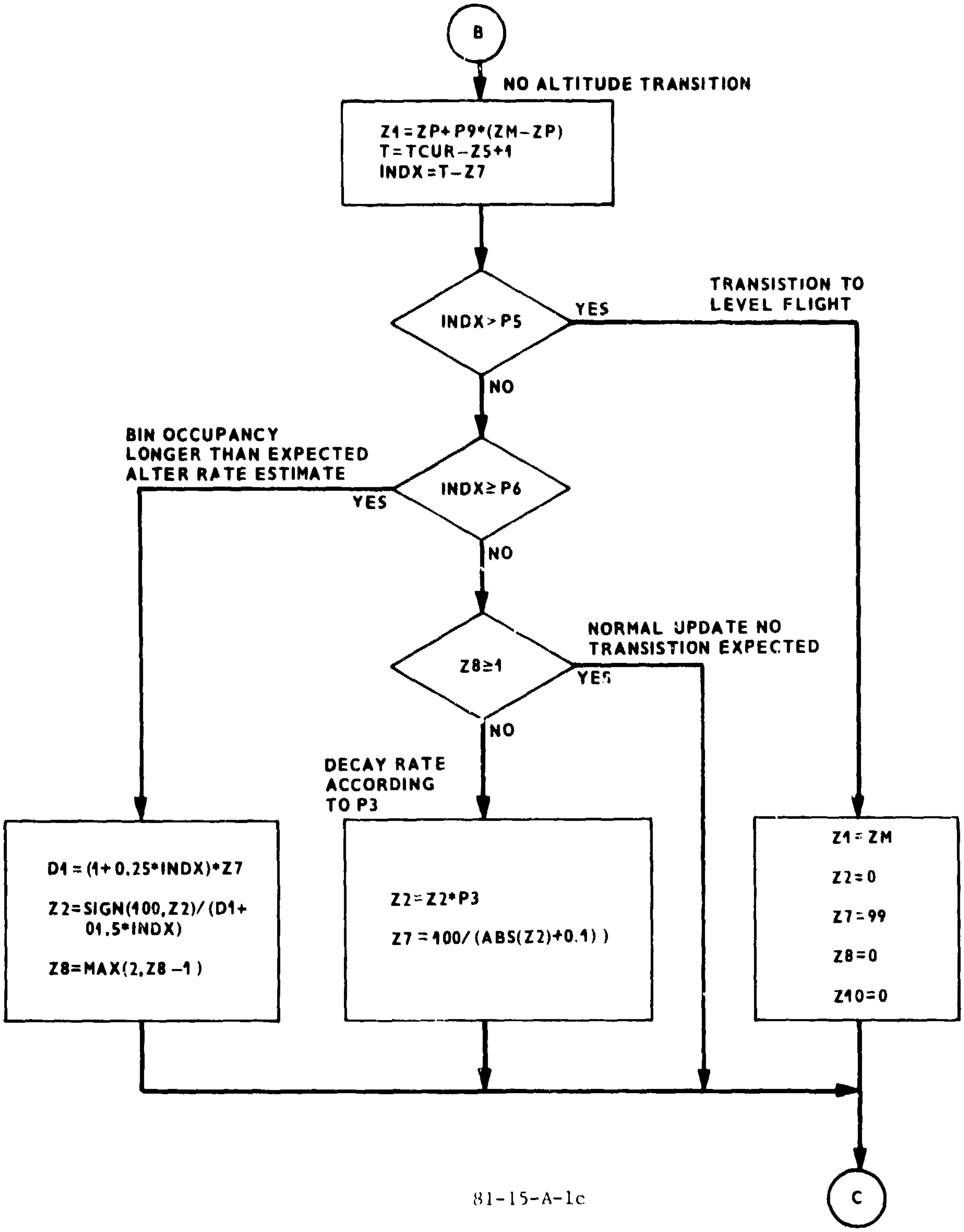

FIGURE A-1. NONLINEAR TRACKING LOGIC AS INTEGRATED INTO TRIACT (Sheet 3 of 3)

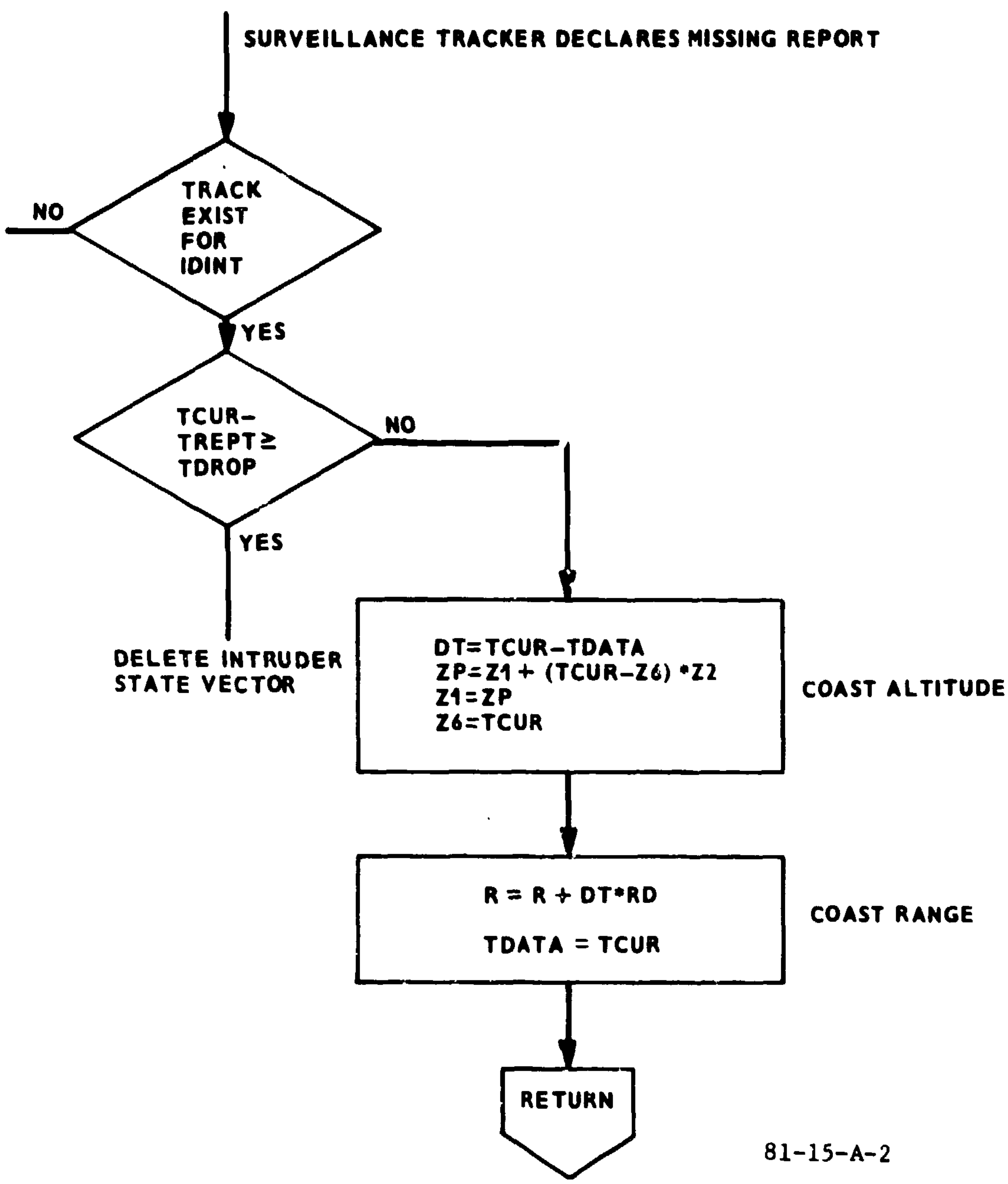

FIGURE A-2. INTEGRATION OF NONLINEAR TRACKING LOGIC INTO MISSING REPORT LOGIC OF TRIACT